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Brief paper

A state observer for continuous oscillating systems under intrinsic pulse-modulated feedback[☆]

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ABSTRACT

A static gain observer for linear continuous plants with intrinsic pulse-modulated feedback is analyzed. The purpose of the observer is to asymptotically drive the state estimation error to zero and synchronize the sequence of pulse modulation instants estimated by the observer with that of the plant. Conditions on the observer gain matrix locally stabilizing the observer error along an arbitrary periodic plant solution are derived and illustrated by simulation for the case of pulsatile testosterone regulation.

1. Introduction

Continuous dynamics with instant modulated impulses give rise to a broad class of hybrid systems with important applications in e.g. electronics and telecommunications. Since impulse-modulated signals are introduced in engineered systems for control or communication, the generated modulated impulse sequence is typically assumed to be known exactly. Mathematical tools of impulsive control theory are covered in depth in Lakshmikantham, Bainov, and Simeonov (1989) and Samoilenko and Perestyuk (1995).

In biological systems, the mechanism of impulse modulation constitutes e.g. the basis of pulsatile endocrine feedback regulation and underlies the secretion of important hormones such as testosterone, insulin, cortisol, etc. (Evans, Farhy, & Johnson, 2009). In a mathematical model of pulsatile feedback introduced and analyzed in Churilov, Medvedev, and Shepeljavyi (2009), the impulses mark the release instants of certain hormones and communicate the secreted quantities. The impulse control of the endocrine systems is orchestrated by the brain and onerous or impossible to observe in the human for ethical reasons. This poses an observation problem where the hormone concentrations inaccessible for measurement are reconstructed from the available in blood stream hormone measurements. In endocrinology, this problem is routinely resolved by means of deconvolution techniques; see Johnson et al. (2009). Only recently, an observer structure for the estimation of pulsatile hormone release has been suggested and analyzed in Churilov, Medvedev, and Shepeljavyi (2011). There, for a special case of one impulse in the least period of the plant, the conditions under which the observer state estimation error asymptotically converges to zero and the sequence of pulse modulation instances estimated by the observer synchronizes with that of the plant are proved.

The present paper takes further the analysis of Churilov et al. (2011) by considering a general periodic solution to the plant equations with an arbitrary number m of the fired impulses in the least period, i.e. the m-cycle.

The paper is organized as follows. First the equations of the plant with a pulse-modulated feedback and the static feedback observer for it are summarized. The notion of a synchronous mode is introduced describing a situation where the firing instants of

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the observer occur simultaneously with those of the plant. Then a mapping describing the evolution of the observer state vector from one firing time to another is devised and its properties are studied. The conditions under which the observer feedback gain guarantees local asymptotical stability of a synchronous mode with respect to an m-cycle are derived and illustrated by numerical simulations.

2. System equations

Consider a plant governed by the equations

$$\frac{dx}{dt} = Ax + B\xi(t), \qquad z = Cx, \qquad y = Lx \tag{1}$$

where A, B, C, L are real constant matrices of sizes $n_x \times n_x$, $n_x \times 1$, $1 \times n_x$, $n_y \times n_x$, respectively, z is the scalar controlled output, y is the vector measurable output, and x is the state vector. The matrix relationships $CB = 0$, $LB = 0$ apply to (1) and are essential for further analysis. The matrix A is Hurwitz stable and the matrix pair (A, L) is observable. The signal ξ is an intrinsic (non-measurable) pulse-modulated feedback of the controlled output z to the state vector x

$$\xi(t) = \sum_{n=0}^{\infty} \lambda_n \delta(t - t_n), \tag{2}$$

$$t_{n+1} = t_n + T_n, \qquad T_n = \Phi(z(t_n)), \qquad \lambda_n = F(z(t_n)). \tag{3}$$

Here $\delta(\cdot)$ is a Dirac delta function, the time instant t_n is its firing time (frequency modulation) and λ_n represents the corresponding weight (amplitude modulation) (Gelig & Churilov, 1998). The functions $\Phi(\cdot)$ and $F(\cdot)$ are continuous, strictly monotonic and bounded with strictly positive lower bounds.

The states $x(t)$ of system (1)–(3) experience jumps at times $t = t_n$. However, because of the imposed conditions $CB = 0$, $LB = 0$, the outputs $y(t)$, $z(t)$ are continuous.

Notice that system (1)–(3) is hybrid (Matveev & Savkin, 2000). To completely characterize the system dynamics, not only the continuous state vector $x(t)$, but also the discrete time instants t_n should be taken into account, yielding, in fact, an $(n_x + 1)$-dimensional system.

The plant is subject to unknown initial conditions $(x(0), t_0)$ and the first firing instant of the pulsatile feedback occurs after the initial time instant, $t_0 \geqslant 0$. To study the system dynamics, the initial conditions can be specified as $(x(t_0^-), t_0)$. Here the minus in the superscript denotes a left-sided limit. A right-sided limit will be denoted by plus.

As demonstrated in Churilov et al. (2009), the above assumptions imply that all the solutions of system (1)–(3) are bounded and there are no equilibria. This corresponds to the self-sustained oscillations arising in endocrine feedback systems with pulsatile secretion; see Evans et al. (2009).

The purpose of observation in hybrid system (1)–(3) is to produce estimates $(\hat{t}_n, \hat{\lambda}_n)$ which are close (in the sense defined below) to the impulse parameters (t_n, λ_n). The main issue is to ensure asymptotical convergence of the sequence $\{\hat{t}_n\}$ to $\{t_n\}$, i.e. to synchronize the observer impulses with those of the plant.

In order to estimate the state vector of (1), an observer mimicking the dynamics of the plant is introduced as

$$\frac{d\hat{x}}{dt} = A\hat{x} + B\hat{\xi}(t) + K(y - \hat{y}), \qquad \hat{y} = L\hat{x}, \qquad \hat{z} = C\hat{x}, \tag{4}$$

where

$$\hat{\xi}(t) = \sum_{n=0}^{\infty} \hat{\lambda}_n \delta(t - \hat{t}_n), \tag{5}$$

$$\hat{t}_{n+1} = \hat{t}_n + \hat{T}_n, \qquad \hat{T}_n = \Phi(\hat{z}(\hat{t}_n)), \qquad \hat{\lambda}_n = F(\hat{z}(\hat{t}_n)) \tag{6}$$

and K is a static feedback gain chosen such that the matrix $D = A - KL$ is Hurwitz. Without loss of generality, it is assumed that $\hat{t}_0 \geqslant t_0$.

The observer above is not a Luenberger observer since the output estimation error is fed back only to the continuous states of the observer and not to the discrete one.

Since the state vector $x(t)$ undergoes jumps at certain times, the closeness of $x(t)$ and $\hat{x}(t)$ cannot be ensured for all t. Indeed, suppose that $\hat{t}_n$ and t_n are close, but do not coincide exactly, and the vector $\hat{x}(t)$ has jumps at $\hat{t}_n$. Let $t_n < \hat{t}_n$ for definiteness. Then if $t_n < t < \hat{t}_n$, the vector $x(t)$ already has a jump, while the vector $\hat{x}(t)$ does not. Thus $x(t)$ and $\hat{x}(t)$ can differ significantly in such time intervals. However, the closeness of $x(t)$ and its estimate $\hat{x}(t)$ can be ensured in the sense that there exists a constant integer $a > 0$ depending on initial conditions and such that $\hat{t}_n - t_{n+a} \to 0$ and $\|\hat{x}(\hat{t}_n^-) - x(t_{n+a}^-)\| \to 0$ as $n \to +\infty$.

Summing up, the overall dynamical system under consideration is the one comprising the plant and the observer and governed by (1)–(3), (4)–(6).

3. The synchronous mode

Let $(x(t), t_n)$ be a solution of plant equations (1)–(3) with the parameters λ_k, T_k, and $x_k = x(t_k^-)$. Suppose that the plant is already running at the moment when the observer is initiated, i.e. $t_a \leqslant \hat{t}_0 < t_{a+1}$ for some $a \geqslant 1$.

Considering the solution $(\hat{x}(t), \hat{t}_n)$ of observer Eqs. (4)–(6) subject to the initial conditions

$$\hat{t}_0 = t_a, \qquad \hat{x}(\hat{t}_0^-) = x(t_a^-),$$

yields

$$\hat{x}_n = x_{n+a}, \qquad \hat{t}_n = t_{n+a}, \qquad \hat{\lambda}_n = \lambda_{n+a}, \quad n = 0, 1, 2, \ldots,$$

and $\hat{x}(t) = x(t)$ for $t \geqslant t_a$. Such a solution $(\hat{x}(t), \hat{t}_n)$ will be called *a synchronous mode* with respect to $(x(t), t_n)$.

An above described synchronous mode will be called *locally asymptotically stable* if for any solution $(\hat{x}(t), \hat{t}_n)$ of (4)–(6) such that the initial estimation errors $|\hat{t}_0 - t_a|$ and $\|\hat{x}(\hat{t}_0^-) - x(t_a^-)\|$ are sufficiently small, it follows that $\hat{t}_n - t_{n+a} \to 0$ and $\|\hat{x}(\hat{t}_n^-) - x(t_{n+a}^-)\| \to 0$ as $n \to \infty$. The latter implies $\hat{\lambda}_n - \lambda_{n+a} \to 0$ as $n \to \infty$.

4. Pointwise mapping and its properties

Pick some solution $x(t)$ of plant equations (1)–(3) with the parameters t_i, λ_i, $i = 0, 1, 2, \ldots$. Further, this solution will be fixed. Consider the pointwise mapping describing the evolution of the observer state:

$$(\hat{x}(\hat{t}_n^-), \hat{t}_n) \mapsto (\hat{x}(\hat{t}_{n+1}^-), \hat{t}_{n+1}). \tag{7}$$

For any real number t and any vector x, select integer numbers k and s, $k \leqslant s$, such that

$$t_k \leqslant t < t_{k+1}, \qquad t_s \leqslant t + \Phi(Cx) < t_{s+1}.$$

Define $P(x, t) = P_{k,s}(x, t)$ with

$$P_{k,s}(x, t) = e^{A(t+\Phi(Cx)-t_s)} x(t_s^+) - e^{D\Phi(Cx)} \big[e^{A(t-t_k)} x(t_k^+) - x - F(Cx)B \big] - \sum_{j=k+1}^{s} \lambda_j e^{D(t+\Phi(Cx)-t_j)} B.$$

For brevity, define $x_k = x(t_k^-)$ and $\hat{x}_n = \hat{x}(\hat{t}_n^-)$.

Theorem 1. *The following statements are true:*

(A) *Pointwise mapping (7) is given by the equations*

$$\hat{x}_{n+1} = P(\hat{x}_n, \hat{t}_n), \qquad \hat{t}_{n+1} = \hat{t}_n + \Phi(C\hat{x}_n). \tag{8}$$