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Brief paper

Frequency-domain identification: An algorithm based on an adaptive rational orthogonal system☆

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ABSTRACT

This paper presents a new adaptive algorithm for frequency-domain identification. The algorithm is related to the rational orthogonal system (Takenaka–Malmquist system). This work is based on an adaptive decomposition algorithm previously proposed for decomposing the Hardy space functions, in which a greedy sequence is obtained according to the maximal selection criterion. We modify the algorithm through necessary changes for system identification.

1. Introduction

System identification concerns the modeling of physical systems that can be described by input–output measurements in the time domain or in the frequency domain. This paper is concerned with the problem of approximating the dynamics of single-input, single-output (SISO) discrete linear time-invariant (LTI) systems that are causal and stable. For the discrete LTI systems considered, let $\{h_k\}$ be the impulse response of the system. Then

$$G(z) = \sum_{l=1}^{+\infty} h_l z^{-l} \tag{1}$$

is the transfer function of the system.

A number of methods have been developed to identify a system. One widely used method is to construct a model structure with a given order, and then to estimate the parameters by the measured data. The most classical and commonly used models include the FIR model, the ARX model, and the ARMAX model. Researchers have been using the rational orthogonal systems by making the model structure a-priori linear in parameters, namely, the transfer function $G(z)$ is approximated by

$$\widetilde{G}(z) = \sum_{l=1}^{n} \theta_l \mathcal{B}_l(z), \tag{2}$$

where $\{\mathcal{B}_l(z)\}$ is a rational orthogonal system, $\{\theta_l\}$ is the n-tuple of parameters to be determined, and n is the order of the model structure. Denote $\theta = [\theta_1\ \theta_2\ \dots\ \theta_n]$ as a parameter vector. Let $\{E_k\}_{k=1}^N$ be the measurements in the frequency domain with

$$E_k = G(e^{j\omega_k}) + v_k, \tag{3}$$

where v_k is the noise; then, using (2), the minimizing parameters θ^* can be determined by a least-squares criterion,

$$\theta^* = \arg\min_{\theta} \frac{1}{N} \sum_{k=1}^{N} |\widetilde{G}(e^{j\omega_k}) - E_k|^2, \tag{4}$$

which can be done almost instantaneously.

The general setting of the rational orthogonal basis is

$$\mathcal{B}_k(z) = \frac{\sqrt{1-|a_k|^2}}{z-a_k} \prod_{l=1}^{k-1} \frac{1-\bar{a}_l z}{z-a_l}, \quad |a_k| < 1. \tag{5}$$

The following are the particular cases that (2) gives rise to with (5).

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- All $a_k = 0$: the classical FIR model.
- All $a_k = a$, a being real valued: the Laguerre models (Mäkilä, 1991; Nurges, 1987; Wahlberg, 1991a; Wahlberg & Mäkilä, 1996).
- All $a_k = a$, a being complex valued: the Kautz models (Wahlberg, 1991b, 1994; Wahlberg & Mäkilä, 1996).

The general setting of the rational orthogonal system (5) was first studied in the 1920s by Takenaka (1925) and Malmquist (1926), and thus was named the Takenaka–Malmquist (TM) system. It has been used in system identification and analyzed in detail since the 1980s by Ninness and other researchers in a large number of publications, including Akçay and Ninness (1999a), Akçay and Ninness (1998), Akçay and Ninness (1999b), Gucht and Bultheel (2003), Ninness (1996), Ninness and Gustafsson (1997), Ninness, Hjalmarsson, and Gustafsson (1997), and Vries and Van den Hof (1998). The study (Oliveira e Silva, 1996) by Olivera e Silva shows that small perturbations in locating the true poles do not induce much error for the approach. Nevertheless, to estimate the true poles of the original system is by no means easy. Our approach is based on a different philosophy. We do not care about where the poles of the true system are for the LTI systems. Using a maximal selection criterion in terms of energy, we find an approximation to the original system in the energy sense through adaptively selected poles defining the rational orthogonal system $\{\mathcal{B}_k\}$.

Here and subsequently, let $\mathbb{D}$ denote the unit disc. With the necessary assumptions on the system, the transfer function (1) belongs to the space of functions holomorphic outside the unit disc. Under the transformation $z \longrightarrow 1/z$, the transfer function turns out to be analytic in $\mathbb{D}$. For uniformity and clarity, the functions considered in this paper are assumed to be analytic inside the unit disc, and they belong to $H_2(\mathbb{D})$ with real-valued impulse response.

For a rational orthogonal system, a well-known and crucial result is as follows.

Theorem 1 (*Akçay and Ninness (1998)*). *Consider the set of functions* $\{ \mathcal{B}_k(z) \}$ *defined by*

$$\mathcal{B}_k(z) = \mathcal{B}_{\{\zeta_1,..\zeta_k\}}(z) \triangleq \frac{\sqrt{1-|\zeta_k|^2}}{1-\overline{\zeta}_k z}\prod_{l=1}^{k-1}\frac{z-\zeta_l}{1-\overline{\zeta}_l z}, \tag{6}$$

where $\zeta_k \in \mathbb{D}$ *and* $k = 1, \ldots$. *Then the set* $X = span\{\mathcal{B}_k(z)\}_{k\ge 1}$ *is complete in* $A(\mathbb{D})$ *or* $H_p(\mathbb{D})$ *for* $1 \le p < \infty$ *if and only if*

$$\sum_{l=1}^{\infty}(1-|\zeta_l|) = \infty, \tag{7}$$

where $A(\mathbb{D})$ *is the disc algebra* $\{ f : f$ *is analytic in the unit disc* $\mathbb{D}$ *and continuous on* $\overline{\mathbb{D}}$ $\}$, *and* $H_p(\mathbb{D})$ *is the Hardy p space of functions* $f(z)$ *analytic in* $\mathbb{D}$.

The condition of completeness (7) is the so-called Szász condition, which has a long history (Heuberger, Van den Hof & Walhlberg, 2005; Szász, 1953).

This paper is based on the theory presented in Qian and Wang (2010) and Qian (2009). We will concentrate on the frequency-domain system identification. The proposed algorithm is based on the rational orthogonal system (6) through finding $\{\zeta_k\}$ under a maximal selection criterion. We incorporate a technical treatment that makes the approximating rational functions to have real-valued coefficients, which is necessary for systems with real-valued impulse responses.

This paper is arranged as follows. In Section 2, we give the problem setting. A brief introduction of the adaptive decomposition algorithm for $f(z)$ in the Hardy space $H_2(\mathbb{D})$ is given in Section 3. Our main result of this paper is given in Section 4. In Section 5, an example is presented. Conclusions are drawn in Section 6.

2. Problem setting

Frequency-domain identification is based on a set of frequency-domain measurements. This set of data is usually obtained by the sinusoid response or, more accurately, by the correlation method (Heuberger et al., 2005; Ljung, 1999).

Without loss of generality, it is also assumed that a set of frequency-domain measurements $\{E_k\}_{k=1}^N$ is available and that these frequency-domain measurements are obtained from a single-input, single-output (SISO) discrete linear time-invariant (LTI) system $f(z)$ in $H_2(\mathbb{D})$ with real-valued impulse response. We further assume that $f(z)$ can be continuously extended to a region containing the closed unit disc. Under these assumptions, if $f(z)$ is a rational function, its coefficients are all real valued and the poles are outside the closed unit disc.

It is then assumed that the structure of measurements $\{E_k\}_{k=1}^N$ is set up to be

$$E_k = f(e^{-j\omega_k}) + v_k \quad (k = 1, 2, \ldots, N),$$

where $\omega_k = \frac{2\pi(k-1)}{N}$, N is even and $\{v_k\}$ is either a bounded sequence satisfying $|v_k| \le \epsilon$, $\epsilon > 0$, or a zero-mean stochastic process with a bounded covariance function; $f(z)$ is the true function to be approximated. Note that equal spacing is, in fact, unnecessary in the algorithm. We just need the measurements for $\omega \in [0, \pi)$; the remaining ones in the interval $(\pi, 2\pi)$ will be obtained by using the conjugate symmetry of the frequency response data.

Let $X_n = span\{\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n\}$, where the orthonormal system $\{\mathcal{B}_k\}_{k=1}^n$ is defined by (6) and $\zeta_k \in \mathbb{D}$. The identification problem that we consider now can be stated as follows.

Frequency-domain identification problem. Given a set of frequency-domain measurements $\{E_k\}_{k=1}^N$ for $f \in H_2(\mathbb{D})$, find a projection $f_n(z) \in X_n$ through finding $\{\zeta_k\} \in \mathbb{D}$, called a *greedy sequence*, such that, for each k,

$$\zeta_k = \arg\max\{|\langle f, B_{\{\zeta_1,..,\zeta_{k-1},\zeta\}}\rangle|^2, \zeta \in \mathbb{D}\}. \tag{8}$$

This is a consecutive energy approximation, meaning that

$$f_{n+1} = f_n + \langle f, \mathcal{B}_{n+1}\rangle \mathcal{B}_{n+1}, \tag{9}$$

and, in the H_2-norm convergence of f_n to f,

$$f = \lim_{n\to\infty} f_n. \tag{10}$$

Such function decomposition is called adaptive Fourier decomposition (AFD), in which it is expected that a sequence $\{\zeta_k\}_{k=1}^n$ is found to give rise to a TM sequence $\{\mathcal{B}_k(z)\}$ that offers efficient approximation to the given function.

3. Adaptive Fourier decomposition for $H_2(\mathbb{D})$ functions

In this section, we provide a brief introduction to the adaptive Fourier decomposition algorithm for $H_2(\mathbb{D})$ functions (Qian, 2009; Qian & Wang, 2010).

The inner product in $H_2(\mathbb{D})$ is defined by

$$\langle f, g\rangle = \frac{1}{2\pi}\int_0^{2\pi} f(e^{j\omega})\overline{g(e^{j\omega})}d\omega.$$

Denote $e_{\{\zeta\}}(z) = \frac{\sqrt{1-|\zeta|^2}}{1-\overline{\zeta}z}$, called *the evacuator at* ζ; $\mathcal{D} = \{e_\zeta, \zeta \in \mathbb{D}\}$ is the dictionary. Using the Cauchy integral formula gives rise to the evaluating functional