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Brief paper

Conditions when minimum variance control is the optimal experiment for identifying a minimum variance controller☆

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ABSTRACT

It is well known that if we intend to use a minimum variance control strategy, which is designed based on a model obtained from an identification experiment, the best experiment which can be performed on the system to determine such a model (subject to output power constraints, or for some specific model structures) is to use the true minimum variance controller. This result has been derived under several circumstances, first using asymptotic (in model order) variance expressions but also more recently for ARMAX models of finite order. In this paper we re-approach this problem using a recently developed expression for the variance of parametric frequency function estimates. This allows a geometric analysis of the problem and the generalization of the aforementioned finite model order ARMAX results to general linear model structures.

1. Introduction

Research in experiment design has been substantial both in the statistical literature (Fedorov, 1972; Pukelsheim, 1993; Silvey, 1980) and in engineering (Goodwin & Payne, 1977; Jansson & Hjalmarsson, 2005; Mehra, 1974; Zarrop, 1979). In particular, in the field of system identification the importance of tailoring the experiment for the particular application for which the model will be used, e.g., prediction, control or simulation (Gevers & Ljung, 1986; Ljung, 1999), has been well known for a long time. This gave rise to the area of system identification for control (Gevers, 2005; Hjalmarsson, 2005). One key contribution from this area has been the observation that it is beneficial from a bias error point of view to perform the identification under the desired operating conditions. Recently it has been shown that this also has advantages from a variance error point of view (Hjalmarsson, 2009).

Minimum variance (MV) control has been a recurrently studied design technique. The principle is important from an industrial perspective as smaller process variations allow one to keep set-points closer to constraints with an increased yield as result. However, the design is also known to be non-robust (Åström & Wittenmark, 1984), partly due to its aggressive nature, and detuning, e.g. by penalizing input changes, is typically necessary. It has been established in the literature that under several conditions, the model to be used for the MV design of a linear time-invariant (LTI) system should be identified in closed loop, using the minimum variance controller of the true system during the estimation stage. This was first established under an output power constraint for models of large orders (Forssell & Ljung, 2000; Gevers & Ljung, 1986; Hjalmarsson & Gevers, 1996). More recently, an important relaxation of the conditions was derived in Hildebrand and Solari (2007) where it was shown that this also holds for ARMAX models of finite order (subject to some degree and factorization conditions) under general input–output power constraints.

Notice that the optimality of the minimum variance controller does not hold for all situations. For instance, it has been established in Agüero and Goodwin (2007) that for Box–Jenkins models, the best experiment to be applied for identification purposes, under an input power constraint, is an open loop experiment for a very general class of cost functions (which includes the one involved in the problem of designing a minimum variance controller).

In this paper we re-approach the problem of designing an experiment for the purpose of constructing a minimum variance controller. To this end, we utilize a recently developed expression for the asymptotic (in sample size) variance of a finite model order frequency function estimate (see Mårtensson (2007) and Ninness and Hjalmarsson (2004)). This expression allows us to analyze the problem in a geometric framework. We re-derive and generalize some results from Hildebrand and Solari (2007) in a transparent

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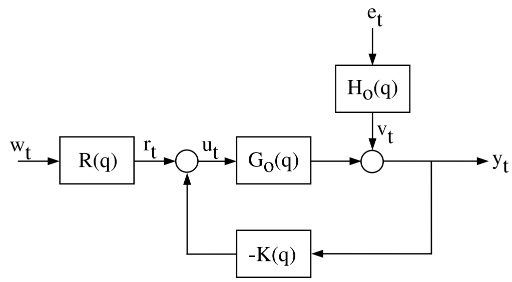

Fig. 1. Block diagram of SISO LTI system with output feedback.

way. Initial results in this direction have appeared in Mårtensson, Rojas, and Hjalmarsson (2009) for ARMAX models.[1]

This paper is structured as follows. In Section 2 we present the system and model assumptions as well as the variance expressions on which we will base our analysis. Section 3 develops the core results of the paper, related to the problem of experiment design for the design of minimum variance controllers. Finally, Section 4 gives some conclusions.

2. Preliminaries

In this section we present the assumptions and mathematical preliminaries which are necessary for developing the results of Section 3.

2.1. System and model

Assumption 2.1. The true system is given by the rational single-input single-output (SISO) LTI minimum phase system $G_o(\mathrm{q})$ (q is the forward shift operator) depicted in Fig. 1 where u_t and y_t represent the measured input and output, respectively, where $\{e_t\}$ and $\{w_t\}$ are zero mean white noise sequences with variance λ_o and 1, respectively, and bounded moments of order $4+\delta$ for some $\delta > 0$. The LTI filter R represents the rational stable minimum phase spectral factor of the reference signal r_t, and H_o is a rational inversely stable LTI filter that is normalized to be monic, i.e., $\lim_{z\to\infty} H_o(z) = 1$. The system G_o includes exactly one time delay. □

Next, we introduce a general family of model structures that will be covered by our framework.

Assumption 2.2. The system is modelled by

$$y_t = T(\mathrm{q},\theta)\chi_t \tag{1}$$

where $T(\mathrm{q},\theta) = [G(\mathrm{q},\theta), H(\mathrm{q},\theta)]$ is an LTI model of the system and the noise dynamics parameterized by the vector $\theta \in \mathbb{R}^n$, and where $\chi_t = [u_t, e_t]^{\mathrm{T}}$.

The model parameterization is such that the *true* system is in the model set, that is, there is a parameter θ^o such that

$$G_o(\mathrm{q}) = G(\mathrm{q},\theta^o), \qquad H_o(\mathrm{q}) = H(\mathrm{q},\theta^o).$$

Finally, following the definitions in Ljung (1999), the model structure is uniformly stable and globally identifiable at θ^o. □

More precise results will be obtained using the following model parameterization.

[1] Unfortunately, there seems to be an error in that paper. The order condition in (ii′) in Theorem 3.1 should be complemented with $n_c \geq n_x + n_b$ where n_x is the order of the polynomial X.

Assumption 2.3 (*Polynomial Models*). The system and noise dynamics are given by

$$G(\mathrm{q},\theta) = \frac{B(\mathrm{q})}{A(\mathrm{q})F(\mathrm{q})} \qquad H(\mathrm{q},\theta) = \frac{C(\mathrm{q})}{A(\mathrm{q})D(\mathrm{q})} \tag{2}$$

where

$$A(\mathrm{q}) = 1 + a_1\mathrm{q}^{-1} + \cdots + a_{n_a}\mathrm{q}^{-n_a}$$
$$B(\mathrm{q}) = b_1\mathrm{q}^{-1} + \cdots + b_{n_b}\mathrm{q}^{-n_b}$$
$$C(\mathrm{q}) = 1 + c_1\mathrm{q}^{-1} + \cdots + c_{n_c}\mathrm{q}^{-n_c}$$
$$D(\mathrm{q}) = 1 + d_1\mathrm{q}^{-1} + \cdots + d_{n_d}\mathrm{q}^{-n_d}$$
$$F(\mathrm{q}) = 1 + f_1\mathrm{q}^{-1} + \cdots + f_{n_f}\mathrm{q}^{-n_f}.$$

The parameter vector θ consists of the coefficients of the polynomials A–F ordered according to increasing indices and in the order C, D, A, B and F.

The type of model described above includes all standard black-box model structures such as ARMAX, output error and Box–Jenkins. The true polynomials will be denoted by A_o etc., and the true polynomial orders correspondingly by n_a^o and so on. Coprimeness holds for B_o and F_o as well as for C_o and D_o. The total number of estimated parameters is in this case $n = n_a + n_b + n_c + n_d + n_f$.

2.2. Asymptotic covariance matrix for the parameters

We will assume that the parameter vector θ is estimated using prediction error identification (Ljung, 1999). Then under Assumptions 2.1 and 2.2, and the additional condition that the entire system is internally stabilized by the LTI controller K, the parameter estimate, which we denote by $\hat{\theta}_N \in \mathbb{R}^n$, has the property that the (normalized) model error $\sqrt{N}(\hat{\theta}_N - \theta^o)$ becomes normal distributed as the sample size N of the data set grows to infinity

$$\sqrt{N}(\hat{\theta}_N - \theta^o) \in \mathrm{As}\mathcal{N}(0, \mathrm{AsCov}\,\hat{\theta}_N). \tag{3}$$

The asymptotic covariance matrix $\mathrm{AsCov}\,\hat{\theta}_N$ of the limit distribution is a measure of the model accuracy. This is reinforced by that, under mild conditions (Ljung, 1999),

$$\lim_{N\to\infty} N \cdot \mathbf{E}[(\hat{\theta}_N - \mathbf{E}\hat{\theta}_N)^{\mathrm{T}}(\hat{\theta}_N - \mathbf{E}\hat{\theta}_N)] = \mathrm{AsCov}\,\hat{\theta}_N.$$

Under Assumptions 2.1 and 2.2, the asymptotic covariance matrix $\mathrm{AsCov}\,\hat{\theta}_N$ obtained in prediction error identification can be written as

$$\mathrm{AsCov}\,\hat{\theta}_N = \langle \Psi, \Psi \rangle^{-1} \tag{4}$$

where $\Psi : \mathbb{C} \to \mathbb{C}^{n\times 2}$ is the gradient with respect to the estimated parameters of the one-step ahead predictor, normalized by the inverse of the noise standard deviation, and where the Gramian $\langle \Psi, \Psi \rangle$ is given by the integral

$$\langle \Psi, \Psi \rangle \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} \Psi(\mathrm{e}^{\mathrm{j}\omega})\Psi^*(\mathrm{e}^{\mathrm{j}\omega})\mathrm{d}\omega \tag{5}$$

(superscript * denotes complex conjugate transpose).

By expressing the signal pair $\chi_t = [u_t\ e_t]^{\mathrm{T}}$ in terms of $\xi_t = [w_t\ e_t]^{\mathrm{T}}$, which has a stable spectral factor given by

$$R_\chi \triangleq \begin{bmatrix} S_o R & -KS_oH_o \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\lambda_o} \end{bmatrix} \tag{6}$$

where $S_o(\mathrm{q}) = 1/(1 + K(\mathrm{q})G_o(\mathrm{q}))$ is the closed loop sensitivity function, the prediction error gradient is given by

$$\Psi(z) = T'(z,\theta^o)R_{\mathrm{SNR}}(z) \tag{7}$$