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Conditions for stabilization of the tokamak plasma vertical instability using only a massless plasma analysis☆

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ABSTRACT

This paper describes the problem of feedback control for stabilization of the plasma vertical instability in a tokamak. Such controllers are typically designed based on a model that assumes the plasma mass m is identically zero while in reality the mass is small but positive. The assumption that m is zero can lead to a controller that appears to be stabilizing according to the massless analysis but in fact can increase the instability of the physical system.

In this work, we consider a general class of controllers, which contains as a special case the type of controller most commonly used in operating tokamaks to stabilize the vertical instability, a proportional-derivative controller. Suppose C is a controller in this class which stabilizes the vertical instability with plasma mass assumed to be zero. We give easy-to-check necessary and sufficient conditions for C to also stabilize the physical system, in which the plasma actually has a small mass. We allow for the possibility that the tokamak could have both superconducting and resistive conductors.

The practical implications of the results presented provide substantial insight into some long-standing issues regarding feedback stabilization of the vertical instability with PD controllers and also provide a rigorous foundation for the common practice of designing controllers assuming $m = 0$. For controllers that operate only on the plasma vertical position, we settle the question: when are $m = 0$ models predictive of actual plasma behavior?

1. Introduction

This paper considers the problem of feedback control for stabilization of the vertical instability in tokamaks. Tokamaks are torus-shaped devices designed to confine a plasma composed of ionized hydrogen isotopes while the plasma is heated to initiate fusion reactions. An example is shown in Fig. 1, which illustrates the KSTAR tokamak in Daejon, Korea (Oh et al., 2008). Introductory descriptions of tokamaks and associated plasma control problems are provided in Pironti and Walker (2005) and Pironti and Walker (2006).

A cross-section of the KSTAR device is shown in Fig. 2, with a plasma cross-section shown in the interior. The instability we consider is one in which the toroidal plasma moves either up or down in the vacuum chamber until it meets the interior vessel wall and is extinguished. One or more of the control coils is typically connected in feedback with a measurement of the vertical position to provide stabilizing control. Currents induced in control coils and passive conductors by the plasma motion provide damping, but cannot actually stabilize the instability. In KSTAR, the active control coils 1 through 14 outside of the vacuum vessel are superconducting and are used to establish the plasma equilibrium. The internal coils 15 through 18 are copper, with coils 15 and 17 dedicated to vertical position (stability) control and coils 16 and 18 used for radial position control.

Feedback controllers for stabilizing the vertical instability in operating tokamaks are almost always designed based on a massless model of the plasma. However, the plasma does in fact have a small positive mass, and of all the controllers C which stabilize the mass zero plasma, some stabilize the $m > 0$ plasma (at least for small $m > 0$) and others do not. As we shall see this is a bifurcation-like phenomenon, with our goal being to not have C on an $m > 0$ destabilizing branch. We provide inequalities saying exactly when this does or does not happen (see Section 1.3). An

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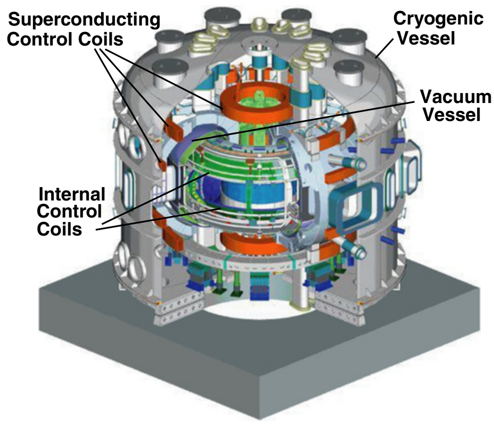

Fig. 1. Three-dimensional view illustration of the KSTAR tokamak.

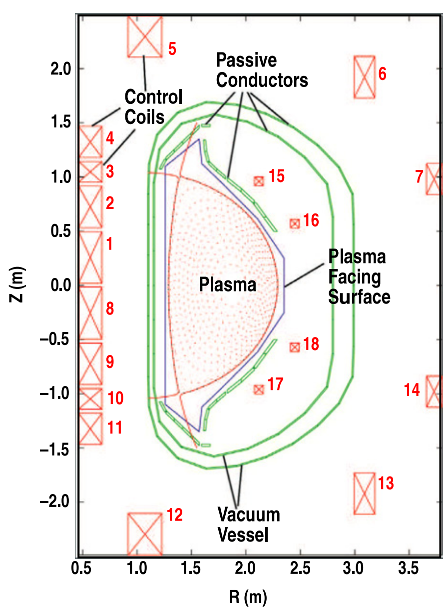

Fig. 2. Cross-section of the KSTAR tokamak.

illustration is Example 1.1 with a PD controller, for which choosing an $m > 0$ stabilizing controller amounts to its gains having the correct sign.

Our results here work under general hypotheses which apply to a far broader class than the PD controllers currently found in today's tokamaks. Thus they resolve to a significant extent the massless vs massive model issue when future tokamaks routinely deploy more sophisticated control algorithms (e.g. LQG or H^∞). For example, confirmation of the correct inequality required for stabilizing gains (i.e. with correct sign) using physics intuition, as is presently done for PD controllers, is much more difficult in high order multivariable controllers.

We emphasize that the primary purpose of this work is not to perform a conventional robustness analysis of a particular form of controller. Rather, the primary purpose is to define conditions under which the zero-mass model can be used at all to develop controllers for stabilizing the vertical instability of the physical plasma-in-tokamak system. These conditions are of inherent theoretical interest, and also have some practical implications for plasma control design.

We point out that these results for tokamaks are based on a mathematical theorem with a clean statement (Theorem 2.7). Thus it might be possible to use this theorem to analyze control of other physical systems having a mass near zero.

1.1. Background on the model

The dynamic description of the plant comprising a tokamak confining an assumed axisymmetric plasma is constructed from the basic electromagnetic equation (Walker & Humphreys, 2006)

$$M\delta\dot{I} + R\delta I + \Psi_z\dot{z}_C + \Psi_r\dot{r}_C = U\delta V \tag{1}$$

where M and R are the mutual inductance and resistance of the toroidal conductors whose currents define the states of (1), and Ψ_z, Ψ_r represent the partial derivatives of flux values at those conductors with respect to vertical (z_C) and radial (r_C) motion of the plasma current centroid ("center of mass" of the distributed plasma current). Toroidal currents in (respectively, voltages on) conductors are represented by the vector I (resp., V) while $\delta I = I - I_{\text{eq}}$ (resp., $\delta V = V - V_{\text{eq}}$) represents a perturbation of the currents (voltages) from their values defining a nominal plasma equilibrium. The vector I includes both currents in active control coils and in toroidal conducting vessel elements. In the following, we use the notation $\delta I = [\delta I_c\ \delta I_v]^T$ to represent a partitioning of the current vector into the n_c active control coils and the n_v passive (vacuum vessel) currents, $U = [\mathbf{I}_{n_c}\ \mathbf{0}_{n_c\times n_v}]^T$, where $\mathbf{I}_{n_c}$ and $\mathbf{0}_{n_c\times n_v}$ are identity and zero matrices respectively.

The motion of the current centroid for a plasma having mass m can be represented by the inertial momentum equations

$$m\ddot{z}_C = f_z\delta z_C + f_I\delta I \tag{2}$$

$$m\ddot{r}_C = f_r\delta r_C + (\partial F_r/\partial I)\delta I \tag{3}$$

where $\delta z_C = z_C - z_{C,\text{eq}}$, $\delta r_C = r_C - r_{C,\text{eq}}$ represent perturbed values of plasma current centroid vertical and radial coordinates relative to their values at the nominal plasma equilibrium, $f_z = \partial F_z/\partial z_C$, $f_I = \partial F_z/\partial I$, $f_r = \partial F_r/\partial r_C$, and F_z, F_r are the total vertical and radial forces on the plasma, all quantities derived from a linearization of the plasma response around the chosen nominal plasma equilibrium. We note that $\Psi_z = f_I^T$ (Ambrosino & Albanese, 2005; Walker & Humphreys, 2009). The mass $m > 0$, which is difficult to accurately estimate, will vary slowly relative to the dynamics of the vertical stability, and therefore may be considered as (an unknown) constant in the analysis presented here.

Eqs. (1) through (3) can be combined to form the overall plant model. From Eq. (1) we obtain

$$M_\#\delta\dot{I} + R\delta I + \Psi_z\dot{z}_C = U\delta V \tag{4}$$

where $M_\# = M + \Psi_r(\partial r_C/\partial I)$ and $\partial r_C/\partial I$ is computed from (3) after setting $m = 0$. (Justification for setting $m = 0$ for radial response but not for vertical response is discussed below). Defining the variables $v_z = \dot{z}_C = \mathrm{d}(\delta z_C)/\mathrm{d}t$, $x_z = [v_z^T\ \delta z_C^T]^T$, we can write (2) as

$$\begin{pmatrix} 0 & 1 \\ m & 0 \end{pmatrix}\dot{x}_z + \begin{pmatrix} -1 & 0 \\ 0 & -f_z \end{pmatrix}x_z + \begin{pmatrix} 0 \\ -f_I \end{pmatrix}\delta I = 0.$$

Combining with (4), we obtain the matrix equation

$$\tilde{M}\dot{x} + \tilde{R}x = \tilde{U}\delta V, \tag{5}$$

$$x = \begin{pmatrix} v_z \\ \delta z_C \\ \delta I \end{pmatrix}; \qquad \tilde{M} = \begin{pmatrix} 0 & 1 & 0 \\ m & 0 & 0 \\ 0 & \Psi_z & M_\# \end{pmatrix}; \tag{6}$$

$$\tilde{R} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -f_z & -f_I \\ 0 & 0 & R \end{pmatrix}; \qquad \tilde{U} = \begin{pmatrix} 0 \\ 0 \\ U \end{pmatrix}.$$

If the equilibrium plasma boundary is sufficiently vertically elongated, so that $f_z > 0$, it can be shown Walker and Humphreys (2009) that the system (5) possesses a single positive real eigenvalue. The eigenvector corresponding to the unstable root corresponds to a nearly rigid vertical motion of the plasma