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Direct data-driven filter design for uncertain LTI systems with bounded noise[☆]

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ABSTRACT

This paper investigates the filter design problem for linear time-invariant dynamic systems when no mathematical model is available, but a set of initial experiments can be performed where also the variable to be estimated is measured. Instead of using the initial experimental data to identify a model on the basis of which a filter is designed, these data are used to directly design a filter. Assuming norm-bounded disturbances and noises, a Set Membership formulation is followed. For classes of filters with exponentially decaying impulse response, approximating sets are determined that guarantee to contain all the solutions to the optimal filtering problem, where the aim is the minimization of the induced norm from disturbances to the estimation error. A method is proposed for designing almost-optimal linear filters with finite impulse response, whose worst-case filtering error is at most twice the lowest achievable one. In the $\mathcal{H}_\infty$ SISO case, an efficient technique is presented, that allows the evaluation of bounds on the guaranteed worst-case filtering error of the designed filter. Numerical examples illustrate the effectiveness of the proposed solution.

1. Introduction

This paper proposes a novel way to design a filter that, operating on the measured output of a linear time-invariant (LTI) dynamic system, gives a (possibly optimal in some sense) estimate of some variable of interest. In particular, a discrete-time, finite dimensional, LTI dynamic system S is considered, for example described in state-space form as:

$$x^{t+1} = Ax^t + Bw^t$$
$$\tilde{y}^t = C_1x^t + Dw^t$$
$$z^t = C_2x^t$$

where, for a given time instant $t \in \mathbb{N}$: $x^t \in \mathbb{R}^n$ is the unknown system state; $\tilde{y}^t \in \mathbb{R}^{n_y}$ is the known system output, measured by noisy sensors; $z^t \in \mathbb{R}^{n_z}$ is the variable to be estimated; $w^t \in \mathbb{R}^{n_w}$ is an unknown multivariate signal that collects all the process disturbances and measurement noises affecting the system; A, B, C_1, C_2 and D are constant matrices of suitable finite dimensions.

Such an estimation problem has been extensively investigated in the literature over the last five decades and plays a crucial role in control systems and signal processing. At the beginning, a stochastic approach was followed and the standard Kalman filter theory was derived, see e.g. Anderson and Moore (1979) and Gelb (1974). Later, the subject of worst-case filtering was treated and the well-established $\mathcal{H}_\infty$, $\mathcal{H}_2$ and ℓ_1 approaches were developed, see e.g. Grigoriadis and Watson Jr. (1997), Shaked and Theodor (1992), Voulgaris (1995b) and the references therein.

The previously mentioned methodologies relied initially on the exact knowledge of the process S under consideration, and later were extended to uncertain systems, thus leading to the so-called robust filtering techniques. These works substantially follow a model-based approach, assuming systems with state-space descriptions possibly affected by norm-bounded or polytopic uncertainties in the system matrices or uncertainties described by integral quadratic constraints, see e.g. Duan, Zhang, Zhang, and Mosca (2006), Xie, Lu, Zhang, and Zhang (2004) and the references therein.

When the process is not completely known, a data-driven approach to the filtering problem is usually obtained by adopting a two-step procedure:

(1) An approximate model of the process is identified from prior information (physical laws, ...), making use of a sufficiently informative noisy data set.
(2) On the basis of the identified model, a filter is designed whose output is an estimate of the variable of interest.

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It has to be remarked that in step 1, except for peculiar cases (i.e., C_2 actually known), in general measurements not only of $\tilde{y}$ but also of z are needed, in order to have a sufficiently informative data set.

For such general situations, an alternative data-driven approach has been proposed in Milanese, Novara, Hsu, and Poolla (2006, 2009), where the data needed in step 1 of the two-step procedure are used to directly design the filter, thus avoiding the model identification. In Milanese et al. (2006), the advantages of such a direct design approach with respect to the two-step procedure have been put in evidence within a stochastic framework, assuming as optimality criterion the minimization of the estimation error variance. It has been proven that even in the most favorable situations, e.g., if S is stable and no modeling errors occur, the filter designed through a two-step procedure performs no better than the directly designed filter. Moreover, in the case of no modeling errors, the directly designed filter is optimal even if S is unstable, while this is not guaranteed by the two-step filter. More importantly, in the presence of modeling errors, the directly designed filter, although not optimal, is the minimum variance estimator among the selected filter class. A similar result is not assured by the two-step design, whose performance deterioration caused by modeling errors may be significantly larger. In Milanese et al. (2009), the direct design approach is investigated in a nonlinear Set Membership setting, considering as optimality criterion the minimization of the worst-case estimation error. Under some prior assumptions, directly designed filters in nonlinear regression form are derived that not only give bounded estimation errors, but are almost-optimal, i.e., have guaranteed estimation errors not greater than twice the minimum achievable ones.

In this paper, a new direct design approach is investigated, following the research line of Milanese, Ruiz, and Taragna (2007b), where the filtering problem is formulated within a linear Set Membership framework. Assuming norm-bounded disturbances and noises, for classes of filters with exponentially decaying impulse response, approximating sets that guarantee to contain all the solutions to the optimal filtering problem are determined, considering experimental data only. The aim is the minimization of the worst-case gain from the signal w to the estimation error, measured in ℓ_p- and ℓ_q-norm respectively ($1 \le p, q \le \infty$), thus allowing to include as particular cases different well-known problems like $\mathcal{H}_\infty$, $\mathcal{H}_2$ and ℓ_1 filtering. A linear almost-optimal filter is designed, with guaranteed worst-case performances when applied to new data. The previously listed advantages of the direct design approach over the two-step procedure are still preserved in this case, since the two-step filter design does not guarantee similar optimality properties, due to the discrepancies between the actual process and the identified model. A complete design procedure is developed, allowing the user to tune the filter parameters, in order to achieve the desired estimation performances in terms of worst-case error.

The paper is organized as follows. In Section 2, the filtering problem is defined. In Section 3, the direct data-driven filter design is formulated in a Set Membership context. In Section 4, the almost-optimal filter design technique is developed. In Section 5, some simulation examples illustrate the effectiveness of the proposed technique. Section 6 ends the paper with some final remarks.

2. Preliminaries and problem formulation

In this paper, a deterministic description of disturbances and noises is adopted, considering that the signal w is unknown but bounded in a given ℓ_p-norm, and the aim is to design a filter that provides an estimate of z that minimizes the worst-case gain from w to the estimation error, measured in some ℓ_q-norm. To this purpose, let us recall the definition of ℓ_p-norm for a one-sided discrete-time signal $s = \{s^0, s^1, \ldots\}$, $s^t \in \mathbb{R}^{n_s}$ and $p \in \mathbb{N}$:

$$\|s\|_p \doteq \left[\sum_{t=0}^{\infty}\sum_{i=1}^{n_s}|s_i^t|^p\right]^{\frac{1}{p}}, \quad 1 \le p < \infty$$

$$\|s\|_\infty \doteq \max_{t=0,\ldots,\infty}\ \max_{i=1,\ldots,n_s} |s_i^t|$$

and the (ℓ_q, ℓ_p)-induced norm of a linear operator T:

$$\|T\|_{q,p} = \sup_{\|s\|_p=1} \|T(s)\|_q, \quad p, q \in \mathbb{N}.$$

Without loss of generality, the variable z to be estimated is considered unidimensional in the following. In fact, the case $n_z > 1$ may be dealt with by decoupling the overall filtering problem into n_z independent univariate subproblems.

The following worst-case filtering problem has been considered in the literature.

Optimal worst-case filtering problem (OFP1): given integers p and q, find an optimal filter $G_o \in \mathcal{H}_\infty$ such that the estimate $\hat{z}_{G_o} = G_o(\tilde{y})$ achieves a finite gain

$$\gamma_o^{\mathcal{H}_\infty} = \inf_{G_o \in \mathcal{H}_\infty}\ \sup_{\|w\|_p=1} \|z - \hat{z}_{G_o}\|_q. \quad \square$$

A closed form solution to this problem is available only for certain special cases (see e.g. Hassibi, Sayed, and Kailath, 1996a) and the minimum may be not unique. The following simpler problem has then been investigated, by relaxing the minimization condition and looking for suboptimal solutions.

Suboptimal worst-case filtering problem (SFP1): given a scalar $\gamma > 0$ and integers p and q, find a filter $G \in \mathcal{H}_\infty$ such that the estimate $\hat{z}_G = G(\tilde{y})$ guarantees

$$\sup_{\|w\|_p=1} \|z - \hat{z}_G\|_q \le \gamma. \quad \square$$

The set of all the solutions to this problem is given by:

$$\mathcal{G}(\gamma) = \left\{ G \in \mathcal{H}_\infty : \sup_{\|w\|_p=1} \|z - \hat{z}_G\|_q \le \gamma \right\}$$

and the filtering aim is to determine whether $\mathcal{G}(\gamma)$ is non-empty and to find a filter inside this set. Obviously, $\mathcal{G}(\gamma_o^{\mathcal{H}_\infty})$ is the set of all the OFP1 solutions.

The literature on worst-case filtering is extensive. SFP1 has been investigated for exactly known systems in the following cases: $p = q = 2$, named $\mathcal{H}_\infty$ filtering (see e.g. Colaneri and Ferrante, 2002, Grimble and El Sayed, 1990, Hassibi et al., 1996a, Hassibi, Sayed, and Kailath, 1996b, Shaked and Theodor, 1992, Yaesh and Shaked, 1991); $p = 2$ and $q = \infty$, named generalized $\mathcal{H}_2$ filtering (see e.g. Grigoriadis and Watson Jr., 1997, Watson Jr. and Grigoriadis, 1997); $p = q = \infty$, named ℓ_1 filtering (see e.g. Nagpal, Abedor, and Poolla, 1996, Vincent, Abedor, Nagpal, and Khargonekar, 1996, Voulgaris, 1995a,b). The construction of $\mathcal{G}(\gamma)$ is discussed in the literature for the cases of $\mathcal{H}_\infty$ and $\mathcal{H}_2$ filtering. In particular, $\mathcal{G}(\gamma)$ is used in Halder, Hassibi, and Kailath (1996) to build mixed $\mathcal{H}_2$–$\mathcal{H}_\infty$ filtering strategies.

In this literature, the system S is supposed to be known. In most practical applications, this is not the case and a model of S is typically identified from measurements $\tilde{y}$ and $\tilde{z} = z + v$ collected during an initial experiment of finite length N, where v is an additive noise on z. In the present paper, these initial data $\tilde{y}$ and $\tilde{z}$ are used to directly design a filter that provides an estimate of z using new measurements $\tilde{y}$, with (possibly) minimal estimation error.

In the following, the system S is unknown and initially at rest (i.e., $x^0 = 0$, $w^t = 0\ \forall t < 0$, $w^0 \neq 0$), the dimensions n and