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Brief paper

Componentwise ultimate bound and invariant set computation for switched linear systems[☆]

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ABSTRACT

We present a novel ultimate bound and invariant set computation method for continuous-time switched linear systems with disturbances and arbitrary switching. The proposed method relies on the existence of a transformation that takes all matrices of the switched linear system into a convenient form satisfying certain properties. The method provides ultimate bounds and invariant sets in the form of polyhedral and/or mixed ellipsoidal/polyhedral sets, is completely systematic once the aforementioned transformation is obtained, and provides a new sufficient condition for practical stability. We show that the transformation required by our method can easily be found in the well-known case where the subsystem matrices generate a solvable Lie algebra, and we provide an algorithm to seek such transformation in the general case. An example comparing the bounds obtained by the proposed method with those obtained from a common quadratic Lyapunov function computed via linear matrix inequalities shows a clear advantage of the proposed method in some cases.

1. Introduction

Switched systems are a special type of dynamic systems that combine a finite number of subsystems by means of a switching rule (Liberzon, 2003; Lin & Antsaklis, 2009). Switched systems constitute a convenient description for many systems of practical importance, including many industrial processes, aircraft control, control of mechanical systems in general, and power systems. The stability and stabilizability of switched systems is an area where considerable research effort has been spent in recent years (Decarlo, Branicky, Pettersson, & Lennartson, 2000; Liberzon & Morse, 1999; Lin & Antsaklis, 2009; Shorten, Wirth, Mason, Wulff, & King, 2007). Different stability problems for switched systems arise depending on whether stability should hold for every admissible switching signal (arbitrary switching), for every switching signal within some class (constrained switching) or for a specific switching signal (switching stabilization). This paper focuses on the arbitrary switching case.

In general, most attention has been devoted to analysing or ensuring the asymptotic stability of an equilibrium point for the switched system (Decarlo et al., 2000; Liberzon & Morse, 1999; Lin & Antsaklis, 2009; Shorten et al., 2007). However, there exist numerous reasons why asymptotic stability may be prevented in a realistic setting. One such reason is that switching may be employed to drive the state of the switched system close to a point that is not an equilibrium point of all subsystems (Xu, Zhai, & He, 2008). Another reason is that nonvanishing perturbations (also named persistent disturbances) may act on the system (Khalil, 2002, Chapter 9). When asymptotic stability is not possible, ensuring some type of practical stability such as the ultimate boundedness of the state trajectories becomes important.

Some results have been reported on the practical stability of switched systems. In Su, Abdelwahed, and Neema (2005), a switched discrete-time system is considered where switching is state-dependent and the problem is that of finding controls that steer the state from a set of initial states to a set of "safe" states. Zhang, Chen, Sun, Mastorakis, and Alexsandrov (2008) and Zhang, Lu, Chen, and Mastorakis (2008) address control design to ensure uniform ultimate boundedness for switched linear systems with parametric uncertainties under arbitrary switching by means of a common Lyapunov function approach. In Zhang and Zhao (2002), the authors address the design of both the control and switching strategy to achieve uniform ultimate boundedness of the system state. Most existing ultimate bound computation methods either make use of level sets of a Lyapunov function or employ some

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norm of the system state to compute the ultimate bound set. For switched linear systems, a quadratic Lyapunov function common to all subsystems can be computed via linear matrix inequalities (LMIs) in case it exists (see, for example, Section 4.3 of Shorten et al. (2007) and the references therein).

In this paper, we address the computation of ultimate bounds and invariant sets for switched continuous-time linear systems. We derive a novel computation method that is based on componentwise analysis and extends previous results presented by the authors in Haimovich, Kofman, and Seron (2008) and Kofman, Haimovich, and Seron (2007). The proposed method provides a new sufficient condition for practical stability and relies on the existence of a transformation that takes all matrices of the switched linear system into a form satisfying certain properties. These properties relate to the concept of Metzler matrices and an associated matrix operation (see (1) in the *Notation* subsection). The use of these new tools and comparison-type results based on these tools distinguish the present paper from our previous results for non-switched continuous-time systems (Haimovich et al., 2008; Kofman et al., 2007) and, thus, constitute one of this paper's novel aspects. We show that the transformation required by the proposed method can be found in the well-known case where the subsystem matrices of the switched linear system generate a solvable Lie algebra. More importantly, another contribution of the present paper is to provide an algorithm to seek the desired transformation that is not restricted to the solvable Lie algebra case. Note that obtaining the required transformation in the switched-linear case is a much more difficult task than in the non-switched case treated in Haimovich et al. (2008) and Kofman et al. (2007), where the transformation was simply a change of coordinates to the Jordan canonical form.

Advantages of the proposed method include its complete systematicity and that it requires neither the computation of a Lyapunov function nor the use of a norm for the system state. An interesting feature of the method is that the ultimate bounds obtained are polyhedral if the required transformation is real, and of a mixed polyhedral/ellipsoidal form if the transformation is complex. To illustrate the results, we provide an example where the matrices of the switched linear system do not generate a solvable Lie algebra. We show that the algorithm is able to find the transformation required by our method, which yields ultimate bounds that are tighter than those obtained by means of a common quadratic Lyapunov function computed via LMIs. A preliminary conference version of parts of the results presented here, as well as parallel results for discrete-time switched linear systems, was published in Haimovich and Seron (2009).

The componentwise ultimate-bound computation method of Haimovich et al. (2008) and Kofman et al. (2007) has been successfully applied to the analysis of sampled-data systems with quantisation (Haimovich, Kofman, & Seron, 2007) and to the development of new controller design methods (Kofman, Seron, & Haimovich, 2008). Moreover, a novel application in fault tolerant control systems has been recently reported in, e.g., Olaru, De Doná, and Seron (2008), Seron, Zhuo, De Doná, and Martínez (2008) and Yetendje, Seron, De Doná, and Martínez (2010). In these papers, the method of Kofman et al. (2007) has been employed to obtain invariant sets where the system behaviour under "healthy" and "faulty" operation can be confined; fault tolerance can be achieved whenever those sets are "separated" in some sense. Thus, the results presented in the current paper have relevance in fault tolerant control systems and we envisage their application in the analysis and design of improved strategies with fault tolerance guarantees.

Notation. $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{N}_0$ and $\mathbb{C}$ denote the reals, nonnegative reals, nonnegative integers and complex numbers, respectively, and j the imaginary unit ($\mathrm{j}^2 = -1$). If $x(t)$ is a vector-valued function, then $\limsup_{t\to\infty} x(t)$ denotes the vector obtained by taking $\limsup_{t\to\infty}$ of each component of $x(t)$, and similarly for 'max'. $|M|$, $\mathbb{R}\mathrm{e}(M)$ and $\mathbb{I}\mathrm{m}(M)$ denote the *elementwise* magnitude, real part, and imaginary part, respectively, of a matrix or vector M. The (i, k)-th entry of M is denoted $M_{i,k}$ and its k-th column $(M)_{:,k}$. If $X, Y \in \mathbb{R}^{n\times m}$, the expression '$X \preceq Y$' denotes the set of componentwise inequalities $X_{i,k} \le Y_{i,k}$, $i = 1, \dots, n$, $k = 1, \dots, m$, and similarly for $X \succeq Y$. Given matrices $M_{\ell_1}, M_{\ell_2}, \dots, M_{\ell_n}$, the notation $\left(\prod_{r=\ell_n}^{r=\ell_1} M_r\right)$ denotes the product $M_{\ell_1} M_{\ell_2} \dots M_{\ell_n}$. Given a matrix $M \in \mathbb{C}^{n\times n}$, $\boldsymbol{\rho}(M)$ denotes its spectral radius, that is, the maximum magnitude of its eigenvalues. A matrix $M \in \mathbb{R}^{n\times n}$ is *Metzler* if $M_{i,k} \ge 0$ for all $i \ne k$. M is Metzler if and only if $\mathrm{e}^{Mt} \succeq 0$ for all $t \ge 0$. Given an arbitrary matrix $N \in \mathbb{C}^{n\times n}$, we define $\mathcal{M}(N) \in \mathbb{R}^{n\times n}$ as the matrix whose entries satisfy

$$[\mathcal{M}(N)]_{i,k} = \begin{cases} \mathbb{R}\mathrm{e}\{N_{i,k}\} & \text{if } i = k, \\ |N_{i,k}| & \text{if } i \ne k. \end{cases} \tag{1}$$

Note that $\mathcal{M}(N)$ is Metzler for every $N \in \mathbb{C}^{n\times n}$.

2. Main results

Consider the continuous-time switched system

$$\dot{x}(t) = A_{\sigma(t)} x(t) + E_{\sigma(t)} w(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $w(t) \in \mathbb{R}^p$ is a perturbation, and

$$\sigma : \mathbb{R}_+ \to \{1, 2, \dots, \mathrm{N}\} \tag{3}$$

is the piecewise constant switching function, assumed to have a finite number of discontinuities in every bounded interval. The evolution of the perturbation w is unknown but assumed to have a componentwise bound

$$|w(t)| \preceq \mathbf{w}, \quad \text{for all } t \ge 0, \tag{4}$$

where $\mathbf{w} \in \mathbb{R}_+^p$ is a known constant vector.

Theorem 1 derives transient and ultimate bounds on the switched continuous-time system state that are valid for any realization of the switching function σ and, in addition, can take the componentwise form of the perturbation bound (4) into account. The proof of Theorem 1 is a minor modification of that of Theorem 2 in Haimovich and Seron (2009) and is omitted for the sake of conciseness.

Theorem 1. *Consider the switched system (2) with switching function (3) and componentwise perturbation bound (4). Let $V \in \mathbb{C}^{n\times n}$ be invertible and define*

$$\Lambda_i \triangleq V^{-1} A_i V, \qquad \Lambda \triangleq \max_{i=1,\dots,\mathrm{N}} \mathcal{M}(\Lambda_i), \tag{5}$$

$$\mathbf{z} \triangleq \max_{i=1,2,\dots,\mathrm{N}} \left[\max_{w:|w|\preceq \mathbf{w}} |V^{-1} E_i w| \right] \tag{6}$$

where $\mathcal{M}(\cdot)$ is the operation defined in (1). Suppose that Λ is Hurwitz and define

$$\phi \triangleq \max\{|V^{-1}x(0)|, -\Lambda^{-1}\mathbf{z}\}, \quad \text{and} \quad \eta \triangleq \phi + \Lambda^{-1}\mathbf{z}. \tag{7}$$

Then, the states of system (2)–(4) are bounded as

$$|V^{-1}x(t)| \preceq -\Lambda^{-1}\mathbf{z} + \mathrm{e}^{\Lambda t}\eta, \tag{8}$$

for all $t \ge 0$, and ultimately bounded as

$$\limsup_{t\to\infty} |V^{-1}x(t)| \preceq -\Lambda^{-1}\mathbf{z}. \tag{9}$$

We next present two corollaries which provide, respectively, componentwise bounds and an invariant set for the states of the linear switched system (2)–(4).

Corollary 2. *Under the conditions of Theorem 1, the states of the linear switched continuous-time system (2)–(4) are componentwise*