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Brief paper

Output-feedback finite-time stabilization of disturbed LTI systems[☆]

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ABSTRACT

Semi-global finite-time exact stabilization of linear time-invariant systems with matched disturbances is attained using a dynamic output feedback, provided the system is controllable, strongly observable and the disturbance has a bound affine in the state norm. The novel non-homogeneous high-order sliding-mode control strategy is based on the gain adaptation of both the controller and the differentiator included in the feedback. A robust criterion is developed for the detection of differentiator convergence to turn on the controller at a proper time.

1. Introduction

Robust finite-time stabilization is often required in modern control theory. Traditional (first order) sliding mode (SM) control (Utkin, 1992) features insensitivity to bounded disturbances acting in the control channel (matched disturbances). The recently introduced High-Order SM (HOSM) controllers (Dinuzzo & Ferrara, 2009; Levant, 2001, 2005; Plestan, Glumineau, & Laghrouche, 2008) also allow for the robust finite-time stable output regulation irrespectively to the output relative degree and provide for the chattering attenuation option (Boiko, 2009). HOSM controllers have already found numerous applications; e.g. see Ferrara, Giacomini, and Vecchio (2007), Kunusch, Puleston, Mayosky, and Riera (2009), Pisano and Usai (2004) and Shtessel and Tournes (2009).

HOSM controllers were originally designed for single-input–single-output (SISO) systems. A list of predesigned controllers for each relative degree of the output is produced. The controllers utilize the output derivatives calculated by robust exact finite-time-convergent HOSM differentiators (Levant, 2003).

An extension to the multi-input–multi-output (MIMO) case is obtained in Bartolini, Ferrara, Usai, and Utkin (2000), Defoort, Floquet, Kokosy, and Perruquetti (2009) and Edwards, Floquet, and Spurgeon (2008). The whole state is assumed to be known in Bartolini et al. (2000), and the traditional sign function of the 1st order sliding modes (1-SMs) is replaced with a 2-SM controller (namely, the sub-optimal algorithm). Robust asymptotically stable output regulation is provided, and the whole state is used, not only the output.

The case of a well-defined vector relative degree is considered in Defoort et al. (2009) and Edwards et al. (2008). Thus, the MIMO problem is decomposed into multiple SISO ones, and an output-based controller is developed. Asymptotic stability is ensured in Edwards et al. (2008), while the system has to be BIBS stable with respect to smooth disturbances. Only an output regulation problem is considered in Defoort et al. (2009). The results of Defoort et al. (2009) and Edwards et al. (2008) are only valid for bounded uncertainties, being inapplicable even to linear time-invariant (LTI) systems with linear uncertainties. Moreover, in spite of the finite-time convergence of observers, an important question has never been considered, how to practically detect the observer convergence for its feedback utilization.

The main contributions of this paper proposes (1) practical robust criteria for the differentiator convergence detection; (2) a novel HOSM-based MIMO output-feedback control strategy for semi-global finite-time exact state stabilization of disturbed linear systems. Moreover, the convergence time of the differentiator is shown to be upper-bounded by a homogeneous function of the initial differentiation error. The criterion is used to properly turn on the proposed control. The disturbance is assumed matched

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and bounded by an affine function of the state norm. Both the controller and differentiator gains are adjusted based on the state observation. Once the control is bounded by a linear function of the state norm, the exact state observation is provided independently of the control aim and control law. Since less control effort is applied due to the gain adaptation, the chattering effect is diminished.

Only the conditions of controllability and strong observability (Hautus, 1983) are imposed. The existence of the output vector relative degree is not required. Both controllability and strong observability are necessary assumptions. Indeed, the former is needed to steer the system toward the origin, and the latter is required to recognize the origin proximity in spite of unknown disturbances.

The results are formally only semi-global, for an initial, though possibly very rough, state-norm upper bound is needed to start the observation. The stabilization would be global, if the exact observation were global. The results are illustrated by a simulation example.

2. Problem formulation

Consider the system

$$\dot{x} = Ax + B[u + w], \qquad y = Cx \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, $w \in \mathbb{R}^m$ are the state, control input, measured output and perturbation signals, respectively. The Lebesgue-measurable disturbance $w(t, x)$ is assumed to satisfy a functional bound, and a rough upper bound is supposed to be known for the initial state $x(t_0)$:

$$\|w(t, x)\| \leq W_1\|x\| + W_2, \quad \forall t \geq t_0, \ \forall x \tag{2}$$

$$\|x(0)\| \leq x_0^+, \tag{3}$$

here W_1, W_2, and x_0^+ are known constants. The only real-time information available for control is the output $y(t)$, and the control objective is to stabilize the system at the origin $x = 0$ in finite-time in spite of the disturbance w.

We assume that $\operatorname{rank}(B) = m$, $\operatorname{rank}(C) = p$, the matrix pair (A, B) is controllable and the triplet (A, B, C) is strongly observable (Hautus, 1983). Recall that the latter assumption is a necessary and sufficient condition to reconstruct the state using only output measurements and it is equivalent to the absence of invariant zeros. This means that any input $u(t) + w(t, x(t))$ keeping the identity $y = 0$ during some time interval also keeps $x = 0$ on the same time interval (Hautus, 1983). Obviously all solutions are extendable till $t = \infty$ provided u is bounded by a linear function of the state norm. Solutions to differential equations and inclusions are understood in Filippov's sense (Filippov, 1988).

3. Convergence criterion for HOSM differentiators

Let $f(t) \in \mathbb{R}$ be a function to be differentiated; then the k-th order HOSM differentiator (Levant, 2003) takes the form

$$\begin{aligned}
\dot{z}_0 &= \nu_0 = -\lambda_k L^{\frac{1}{k+1}} |z_0 - f|^{\frac{k}{k+1}} \operatorname{sign}(z_0 - f) + z_1, \\
\dot{z}_1 &= \nu_1 = -\lambda_{k-1} L^{\frac{1}{k}} |z_1 - \nu_0|^{\frac{k-1}{k}} \operatorname{sign}(z_1 - \nu_0) + z_2, \\
&\vdots \\
\dot{z}_{k-1} &= \nu_{k-1} = -\lambda_1 L^{\frac{1}{2}} |z_{k-1} - \nu_{k-2}|^{\frac{1}{2}} \operatorname{sign}(z_{k-1} - \nu_{k-2}) + z_k, \\
\dot{z}_k &= -\lambda_0 L \operatorname{sign}(z_k - \nu_{k-1}),
\end{aligned} \tag{4}$$

where z_i is the estimation of the true derivative $f^{(i)}(t)$. The differentiator provides for the finite-time exact differentiation under ideal conditions of exact measurement in continuous time. The only information needed is an a priori known upper bound L for $|f^{(k+1)}|$. Then an infinite parametric sequence $\{\lambda_i\} > 0$, $i = 0, 1, \ldots, k, \ldots$, is recursively built, which provides for the convergence of the differentiators for each order k. Such parameters are further called proper. In particular, the parameters $\lambda_0 = 1.1, \lambda_1 = 1.5, \lambda_2 = 2, \lambda_3 = 3, \lambda_4 = 5, \lambda_5 = 8$ are proper and enough till the 5-th differentiation order. With discrete sampling the differential equations are replaced by their Euler approximations. This differentiator provides for the best possible asymptotic accuracy in the presence of input noises (Kolmogorov, 1962; Levant, 2003).

The reliable derivatives' estimations are only available after a finite-time transient. Thus, a controller is to be applied after the transient. Until now this procedure was performed by waiting "enough time" to ensure the differentiator convergence. A preferable way though is to evaluate the transient time or to detect the differentiator convergence.

Denote $\sigma_i = (z_i - f_0^{(i)})/L$, $\vec{\sigma} = (\sigma_0, \ldots, \sigma_k)$. Let $N(\vec{\sigma})$ be any positive-definite homogeneous function of $\vec{\sigma}$ of the weight 1 (Levant, 2005). In particular, $N(\vec{\sigma}) = |\sigma_0|^{1/(k+1)} + |\sigma_1|^{1/k} + \cdots + |\sigma_k|$ can be taken.

Theorem 1. *Consider the HOSM differentiator (4) of order k with proper parameters λ_i. Then there exist two constants $0 < \mu_m \leq \mu_M$, such that for every initial time t_0 the convergence time T satisfies the inequality $\mu_m S < T < \mu_M S$ whenever $N(\vec{\sigma}(t_0)) \leq S$.*

Obviously $T = O(L^{-1/(k+1)})$ with large L. Taken differentiator (4) with properly chosen λ_i the constants μ_m, μ_M can be estimated once and forever by simulation.

Theorem 2. *Consider the HOSM differentiator (4) of order k, with proper parameters $\{\lambda_i\}$. Let*

$$f(t) = f_0(t) + \eta(t), \qquad |f_0^{(k+1)}(t)| < L, \qquad |\eta(t)| \leq k_\eta L \xi^{k+1}, \tag{5}$$

where $f_0(t)$ is an unknown basic signal, $\eta(t)$ is a Lebesgue-measurable sampling noise, ξ is a positive parameter. Suppose also that f is sampled with a possibly variable time step $\tau_s > 0$, and $\tau_s \leq k_\tau \xi$, with k_η, k_τ being some positive constants. Then for any positive constants $\gamma_0, \gamma_1, \ldots, \gamma_k$ and any k_f, $0 < k_f < \gamma_0$, there exist $k_\eta, k_\tau, \gamma_t > 0$, such that if the inequality

$$|z_0 - f(t)| \leq k_f L \xi^{k+1} \tag{6}$$

holds at all sampling instants within the time interval of the length $\gamma_t \xi$, then starting from the beginning of this interval the inequalities

$$|z_i - f_0^{(i)}(t)| \leq \gamma_i L \xi^{k-i+1}, \quad i = 0, 1, \ldots, k \tag{7}$$

hold and are kept forever. The transient time estimation from Theorem 1 remains valid with sufficiently small ξ/S.

Obviously, one can arbitrarily increase γ_t and decrease k_f, k_η, k_τ preserving the statement of Theorem 2. In particular, due to Theorem 1, exact estimations can be ensured at the end of the observation time interval in the limit case, when the measurements are exact and continuous, but $\xi > 0$. It is natural to determine a set of constants k_f, k_η and γ_t by simulation of a single differentiator.

4. Finite-time exact state observation of strongly observable linear systems with unknown inputs

Introduce the following notation. Let $y(t) \in \mathbb{R}^p$ be a vector function of the full rank, and $y^{[k]}(t)$ denote the k-th antidifferentiator $y^{[k]}(t) := \int_0^t \int_0^{s_1} \cdots \int_0^{s_{k-1}} y(s_k) ds_k \ldots ds_2 ds_1$ with $y^{[0]}(t) := y(t)$. Let $Y^{[[k]]}(t) := \begin{bmatrix} Y^{[[k-1]]}(t) \\ y^{[k]}(t) \end{bmatrix}$ with $Y^{[[1]]}(t) := \begin{bmatrix} y(t) \\ y^{[1]}(t) \end{bmatrix}$. Recall the following result.

Lemma 1 (*Bejarano & Fridman, 2010*)**.** *Let system (1) be strongly observable with respect to $y(t)$. Then there exists a computable matrix*