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Brief paper

On emulated nonlinear reduced-order observers for networked control systems[☆]

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ABSTRACT

We consider a general class of nonlinear reduced-order observers and show that the global asymptotic convergence of the observation error in the absence of network-induced constraints is maintained for the emulated observer semiglobally and practically (with respect to the maximum allowable transmission interval) when system measurements are sent through a communication channel. Networks governed by a Lyapunov uniformly globally asymptotically stable protocol are investigated. Our results can be used to synthesize various observers for networked control systems for a range of network configurations, as we illustrate it by considering classes of immersion and invariance observers which include the circle-criterion observers.

1. Introduction

Networked control systems (NCS) refer to systems for which communication between the controller and spatially distributed sensors / actuators is ensured via a shared network channel. Due to their great flexibility, low cost and easy maintenance, NCS have become prevalent in many emerging control applications such as drive-by-wire cars and fly-by-wire aircrafts. However, the communication constraints induced by the network imply additional difficulties compared to classical control systems. When considering the observer design problem, one of the main issues is due to the scheduling: only a subset of sensors is allowed to send their data to the observer at the transmission instants. The sporadic and partial availability of system measurements, which are respectively characterized by the *maximum allowable transmission interval (MATI)* and the *scheduling protocol*, requires the development of appropriate observer designs.

A framework for the synthesis of full order observers for nonlinear NCS has been proposed in Postoyan and Nešić (2010a), via an emulation-like approach, that encompasses the methods proposed in Karafyllis and Kravaris (2009); Postoyan, Ahmed-Ali, and Lamnabhi-Lagarrigue (2009) as particular cases. Provided that the continuous-time observer is sufficiently robust to measurement errors, sufficient conditions are given to guarantee the global convergence of the observation error for various in-network processing implementations and Lyapunov uniformly globally exponentially stable (UGES) protocols. Computable MATI bounds are obtained and our results have been applied to derive linear observers (Postoyan & Nešić, 2010a) and high-gain observers (Postoyan, 2009) for NCS for a range of network configurations. This work has then been extended to larger classes of systems and protocols in Postoyan and Nešić (2010b), by allowing Lyapunov UGAS (uniformly globally asymptotically stable) protocols as introduced in (Nešić & Teel, 2004b), and by assuming that all the input-to-state stability (ISS) assumptions in (Postoyan & Nešić, 2010a) hold with nonlinear gains (and not linear gains). As a consequence, the observation error is no longer ensured to converge globally in the presence of network, but semiglobally (and practically) with respect to the MATI. Obtained results have been applied to build circle-criterion observers (Arcak & Kokotović, 2001) for NCS.

In this paper, we study the emulation of reduced-order observers for NCS. To the best of our knowledge, it is the first time that reduced-order observers are built for NCS subject to scheduling. We start by considering a continuous-time observer design which comes from immersion and invariance techniques (Astolfi, Karagiannis, & Ortega, 2008; Karagiannis, Carnevale, & Astolfi, 2008) and which allows one to recover many observer syntheses as mentioned in Karagiannis et al. (2008), such as linear Luenberger observers, high-gain observers (Gauthier, Hammouri, & Othman,

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1992; Khalil & Esfandiari, 1993), observers for linear systems up to an output injection (Jouan, 2003; Krener & Isidori, 1983) and circle-criterion observers (Arcak & Kokotović, 2001) to mention a few. A model is then derived for the observer design for NCS, which is based on a different set of coordinates compared to Postoyan and Nešić (2010a,b). We show that if the continuous-time observer is built to ensure some ISS properties for the observation error while ignoring the network, then this property will be maintained semiglobally and practically w.r.t. the parameter MATI, when the system measurements are sent through a network controlled by a Lyapunov UGAS protocol, under mild conditions. We use the obtained results to build up a class of immersion and invariance observers as well as circle-criterion observers for NCS.

2. Preliminaries

A function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is continuous, zero at zero and strictly increasing and is of class $\mathcal{K}_\infty$ if, in addition, it is unbounded. By extension, for $\gamma : \mathbb{R}^2_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}\mathcal{K}$ if, for any $(s_1, s_2) \in \mathbb{R}^2_{\geq 0}$, $\gamma(s_1, \cdot)$ and $\gamma(\cdot, s_2)$ are of class $\mathcal{K}$. A continuous function $\gamma : \mathbb{R}^2_{\geq 0} \longrightarrow \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}\mathcal{L}$ if for each $t \in \mathbb{R}_{\geq 0}$, $\gamma(\cdot, t)$ is of class $\mathcal{K}$, and, for each $s \geq 0$, $\gamma(s, \cdot)$ is decreasing to zero. Let $C(\mathbb{R}^p, \mathbb{R}^q)$ and $C^1(\mathbb{R}^p, \mathbb{R}^q)$ respectively denote the space of all continuous and continuously differentiable mappings from $\mathbb{R}^p$ to $\mathbb{R}^q$. The Euclidean norm of a vector or a matrix is denoted by $|\cdot|$. Given a measurable, locally essentially bounded signal $f : [t_0, \infty) \to\to \mathbb{R}^n$, we denote its $\mathcal{L}_\infty$ norm as $\|f\|_\infty = \text{ess.}\sup_{\tau \geq t_0} |f(\tau)|$. When $\|f\|_\infty$ is bounded, we write that $f \in \mathcal{L}_\infty$ and we say that $f \in \mathcal{L}_{loc,\infty}$, when for any $t_0 \leq t_1 \leq t_2 < \infty$, $\text{ess.}\sup_{t_2 \geq \tau \geq t_1} |f(\tau)|$ is bounded. For $(x, y) \in \mathbb{R}^{n+m}$, the notation (x, y) stands for $[x^T, y^T]^T$ and $\mathbb{I}$ denotes the identity matrix of appropriate dimensions. A mapping $h(t, z, y) : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n_z+n_y} \to \mathbb{R}^{n_\eta}$ is said to be left-invertible w.r.t. its second argument if there exists a mapping $h^L : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n_\eta+n_y} \to \mathbb{R}^{n_z}$ such that $h^L(t, h(t, z, y), y) = z$ for all t, z, y. Consider the system:

$$\dot{x} = f(t, x, u), \qquad y = h(t, x), \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $y \in \mathbb{R}^{n_y}$ the output and $u \in \mathbb{R}^{n_u}$ the input.

Definition 1. System (1) is said to be *uniformly bounded-input-bounded-state (UBIBS)* with input u if there exist $K, \gamma \in \mathcal{K}$, such that, for any $x_0 \in \mathbb{R}^{n_x}$, $u \in \mathcal{L}_\infty$: $|x(t)| \leq K(|x_0|) + \gamma(\|u\|_\infty)$ for all $t \geq t_0 \geq 0$. When $u \equiv 0$, we say that it is *uniformly globally stable (UGS)*.

Definition 2. System (1) is said to be *uniformly forward complete with input u* if there exist $\nu_1, \nu_2, \nu_3 \in \mathcal{K}$ and $c \in \mathbb{R}_{\geq 0}$ such that, for any $x_0 \in \mathbb{R}^{n_x}$, $u \in \mathcal{L}_\infty$, along solutions to (1): $|x(t)| \leq \nu_1(t - t_0) + \nu_2(|x_0|) + \nu_3(\|u\|_\infty) + c$ for all $t \geq t_0 \geq 0$.

Remark 3. Definition 2 is inspired by Angeli and Sontag (1999) (in particular Corollary 2.3) where forward completeness characterizations are proposed for time-invariant systems.

3. System models

We pursue the emulation-like approach for the observer design for NCS like in Postoyan and Nešić (2010a,b), which is originally inspired by the work of Nešić and Teel (2004a); Walsh, Beldiman, and Bushnell (2001) for the control of NCS. The approach consists in synthesizing an observer while ignoring the communication constraints, afterwards the networked-induced errors are taken into account.

3.1. Immersion and invariance reduced-order observers

We choose to focus on observer designs which come from immersion and invariance techniques (Astolfi et al., 2008; Karagiannis et al., 2008) because a number of observer constructions available in the literature can then be considered in a unified manner. Consider the plant modeled by the following continuous-time equations:

$$\dot{\eta} = f_\eta(t, \eta, y, w), \qquad \dot{y} = f_y(t, \eta, y, w), \tag{2}$$

where $\eta \in \mathbb{R}^{n_\eta}$ is the unmeasured part of the state, $y \in \mathbb{R}^{n_y}$ is the measurable part of the state, $w \in \mathbb{R}^{n_w}$ is an exogenous disturbance input. For the purpose of constructing an observer which estimates η, we may employ a coordinate transformation, thus we define the variable $z = T_P(t, \eta, y) \in \mathbb{R}^{n_z}$ where $n_z \geq n_\eta$ and $T_P \in C^1(\mathbb{R}_{\geq 0} \times \mathbb{R}^{n_\eta+n_y}, \mathbb{R}^{n_z})$ is a left-invertible mapping in its second argument with left-inverse T_P^L. We denote:

$$\dot{z} = f_z(t, z, y, w), \qquad \dot{y} = f_y(t, T_P^L(t, z, y), y, w), \tag{3}$$

where

$$\begin{aligned} f_z(t, z, y, w) = &\frac{\partial T_P}{\partial t}(t, T_P^L(t, z, y), y) \\ &+ \frac{\partial T_P}{\partial \eta}(t, T_P^L(t, z, y), y) f_\eta(t, T_P^L(t, z, y), y, w) \\ &+ \frac{\partial T_P}{\partial y}(t, T_P^L(t, z, y), y) f_y(t, T_P^L(t, z, y), y, w). \end{aligned}$$

The idea is to design a reduced-order observer for system (3) that will provide us with an estimate of z, namely $\bar{z}$. The estimate of η, denoted $\bar{\eta}$, will then be obtained by using $\bar{\eta} = T_P^L(t, \bar{z}, y)$. In a number of cases, n_z is equal to n_η. Nonetheless, it is known that immersing the plant (2) into a system of larger dimension ($n_z > n_\eta$) may help for designing an observer (see Chapter 5 in Astolfi et al. (2008) and the references therein). Now to show the observation error $\eta - \bar{\eta}$ satisfies some ISS properties, we need to assume that T_P guarantees the following condition.

Assumption 4. There exist $\theta_P, \bar{\theta}_P \in \mathcal{K}_\infty$ such that for any $a, \bar{a} \in \mathbb{R}^{n_z}$, $b, \bar{b} \in \mathbb{R}^{n_y}$ and $t \in \mathbb{R}_{\geq 0}$, this holds: $|T_P^L(t, a, b) - T_P^L(t, \bar{a}, \bar{b})| \leq \theta_P(|a - \bar{a}|) + \bar{\theta}_P(|b - \bar{b}|)$.

The reduced-order observer takes the following form:

$$\left.\begin{aligned} \dot{\bar{q}} &= f_{\bar{q}}(t, \bar{q}, y) \\ \bar{z} &= T_O(t, \bar{q}, y) \\ \bar{\eta} &= T_P^L(t, \bar{z}, y) \end{aligned}\right\} \tag{4}$$

where $\bar{q} \in \mathbb{R}^{n_{\bar{q}}}$ is the observer state ($n_{\bar{q}} = n_z$), $\bar{z} \in \mathbb{R}^{n_z}$ is defined through the output map $T_O \in C^1(\mathbb{R}_{\geq 0} \times \mathbb{R}^{n_{\bar{q}}+n_y}, \mathbb{R}^{n_z})$, where T_O is left-invertible w.r.t. its second argument with left-inverse T_O^L, and T_P^L is used to obtain $\bar{\eta} \in \mathbb{R}^{n_\eta}$, the estimate of η. Variable $\bar{z}$ may seem superfluous since $\bar{\eta} = T_P^L(t, T_O(t, \bar{q}, y), y)$ according to (4). Nevertheless, it is sometimes necessary to analyze the dynamics of $z - \bar{z}$ rather than directly those of $\eta - \bar{\eta}$ in order to deduce stability properties for the observation error, as discussed in Remark 1 in Karagiannis et al. (2008).

Beside the immersion and invariance observers developed in Karagiannis et al. (2008) and in Chapter 5 in Astolfi et al. (2008), the observer formulation (4) allows one to consider various reduced-order observer designs available in the literature, such as the circle criterion observers developed in Arcak and Kokotović (2001) as shown below.

Example 5 (*Circle-Criterion Observers (Arcak & Kokotović, 2001)*). Consider the system:

$$\dot{y} = A_1\eta + G_1\gamma(H_1 y + H_2\eta) + \rho_1(y) \tag{5}$$

$$\dot{\eta} = A_2\eta + G_2\gamma(H_1 y + H_2\eta) + \rho_2(y), \tag{6}$$