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Semi-global stabilization of linear time-delay systems with control energy constraint[☆]

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ABSTRACT

This note is concerned with the problem of semi-globally stabilizing a linear system with an input delay and a constraint on the energy of its input. Under the condition of null controllability with vanishing energy, the parametric Lyapunov equation based L_2 low gain feedback is adopted to solve the problem. The proposed approach is applied to the linearized model of the relative motion in the orbit plane of a spacecraft with respect to another spacecraft in a circular orbit around the Earth to validate its effectiveness.

1. Introduction

Linear systems with actuator magnitude saturation have broad engineering background and are difficult to control. This class of systems has been extensively studied in the past several decades, and many control problems have been studied. Among these problems are global stabilization (Kaliora & Astolfi, 2004; Sussmann, Sontag, & Yang, 1994; Teel, 1992), semi-global stabilization (Lin, 1998), finite gain stabilization (Liu, Chitour, & Sontag, 1996), and local stabilization with a maximized domain of attraction (Hu & Lin, 2001).

On the other hand, control of linear systems in the presence of time delays, especially delays in the control input, has also been attracting significant attention for several decades. The delays in the control input arise from a variety of sources, including signal transmission and computation. In fact, the analysis and design of control systems that takes into account delays in the control input is a classical problem, and many related problems have been studied in the literature (see Chen, Gu, & Nett, 1995; Gu & Liu, 2009; Hale, 1977; Zhang, Zhang, & Xie, 2004, and the references therein). Control systems with both input delay and input magnitude saturation have also received much attention in recent years (see, for example, Lin & Fang, 2007; Mazenc, Mondie, & Niculescu, 2003; Tarbouriech & da Silva, 2000; Yakoubi & Chitour, 2007, and the references therein).

Similarly to magnitude constraints, energy constraints are also encountered naturally in practical systems, because any physical system can only be powered with finite energy. However, the problem of controlling energy-constrained systems has not received as much attention as that of controlling magnitude-constrained systems. Only recently has null controllability with vanishing energy been studied in Ichikawa (2008) and Priola and Zabczyk (2003). More recently, under the assumption of null controllability with vanishing energy, as characterized in Ichikawa (2008) and Priola and Zabczyk (2003) and by using L_2 low gain feedback (Zhou, Lin, & Duan, 2011), we solved the semi-global stabilization problem for linear systems with energy constraints on the control inputs.

In the present note, we go a further step beyond (Zhou et al., 2011) by showing that semi-global stabilization of an input-delayed linear system subject to an energy constraint can also be achieved by a special kind of L_2 low gain feedback, namely,

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parametric Lyapunov equation based low gain feedback, provided that the open-loop system is null controllable with vanishing energy in the absence of input delay. By semi-global stabilization we mean that a controller, whose output satisfies an energy constraint, is designed such that the closed-loop system is locally asymptotically stable with its domain of attraction containing an *a priori* given arbitrarily large bounded set of the state space. It is shown that the delay in the control input can be any arbitrarily large finite value. These results complement those existing results in Zhou et al. (2011). The effectiveness of the proposed approach is validated with its application to the linearized model of the relative motion in the orbit plane of a spacecraft with respect to another spacecraft in a circular orbit around the Earth.

2. Problem formulation

Consider a linear system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in \mathbf{R}^n$ and $u(t) \in \mathbf{R}^m$ are, respectively, the state and input vectors. Let $x(t, x_0, u)$ denote the solution of (1) with initial condition x_0 and input u. Denote

$$L_2\left(0, T, \mathbf{R}^m\right) \triangleq \left\{ f : [0, T] \to \mathbf{R}^m \,\middle|\, \int_0^T \|f(t)\|^2 \,\mathrm{d}t < \infty \right\}.$$

We recall the following definition of null controllability with vanishing energy for system (1).

Definition 1 (*Ichikawa, 2008*)**.** System (1) (or the matrix pair (A, B)) is said to be null controllable with vanishing energy (NCVE) if, for each initial $x(0) = x_0$, there exists a sequence of pairs (T_N, u_N), $0 \le T_N < \infty$, $u_N \in L_2(0, T_N, \mathbf{R}^m)$, such that $x(T_N, x_0, u_N) = 0$ and $\lim_{T_N\to\infty} \int_0^{T_N} \|u_N(t)\|^2 \,\mathrm{d}t = 0$.

Roughly speaking, a system is NCVE if, for any initial condition, there exists a control input with an arbitrarily small energy that steers the state of the system to the origin. This class of systems and the relating control problems have many applications in practice. For example, the relative motion of a spacecraft with respect to another spacecraft in a circular orbit around the Earth is described by a nonlinear system whose linearized version is NCVE (Ichikawa, 2008). Certainly, it is important to accomplish a control objective with an arbitrarily small amount of energy expended.

Lemma 1 (*Priola & Zabczyk, 2003*)**.** *Linear system (1) is NCVE if and only if (A, B) is controllable in the ordinary sense and all the eigenvalues of A are located in the closed left-half s-plane.*

In this note, we consider the linear time-delay system

$$\dot{x}(t) = Ax(t) + Bu(t - \tau), \tag{2}$$

where $x(t) \in \mathbf{R}^n$ and $u(t) \in \mathbf{R}^m$ are, respectively, the state and input vectors, and $\tau > 0$ represents the delay in the control input. Throughout this note, we use $\mathscr{C}_{n,\tau} = \mathscr{C}([-\tau, 0], \mathbf{R}^n)$ to denote the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\mathbf{R}^n$ with the topology of uniform convergence, and $x_t \in \mathscr{C}_{n,\tau}$ to denote the restriction of $x(t)$ to the interval $[t-\tau, t]$ translated to $[-\tau, 0]$, that is, $x_t(\theta) = x(t+\theta)$, $\forall \theta \in [-\tau, 0]$. For any $\psi \in \mathscr{C}_{n,\tau}$, we define $\|\psi\|_c = \sup_{\theta\in[-\tau,0]} \|\psi(\theta)\|$. The problem we are interested in is as follows.

Problem 1 (L_2*-Semi-Global Stabilization*)**.** Let $\Omega \subset \mathscr{C}_{n,\tau}$ be a bounded compact set, and let $E > 0$ be a given scalar. Find a control $u \in \mathcal{U}_{\mathrm{E}}(E)$ with

$$\mathcal{U}_{\mathrm{E}}(E) = \left\{ u : [-\tau, \infty) \to \mathbf{R}^m \,\middle|\, \int_{-\tau}^{\infty} \|u(t)\|^2 \,\mathrm{d}t \le E^2 \right\}$$

such that system (2) is asymptotically stable with Ω contained in the domain of attraction.

3. Extension of L_2-vanishment to nonlinear systems

Toward solving Problem 1, we first recall the L_2 low gain feedback approach studied in Zhou et al. (2011). Assume that the matrix $A(\gamma) : [0, 1] \to \mathbf{R}^{n\times n}$ is a continuous matrix function of γ and such that $\lambda(A(\gamma)) \subset \mathbf{C}^-$, $\forall \gamma \in (0, 1]$ and $\lambda(A(0)) \subset \mathbf{C}^{-0} \triangleq \{s : \mathrm{Re}\{s\} \le 0\}$.

Definition 2. Let $S(\gamma) : [0, 1] \to \mathbf{R}^{m\times n}$ and $A(\gamma) : [0, 1] \to \mathbf{R}^{n\times n}$ *be as stated above*. Then $(S, A) = (S(\gamma), A(\gamma))$ is said to be L_2-vanishing if

$$\lim_{\gamma\to 0^+} \left\| S e^{At} \right\|_{L_2} \triangleq \lim_{\gamma\to 0^+} \left(\int_0^\infty \left\| S e^{At} \right\|^2 \mathrm{d}t \right)^{\frac{1}{2}} = 0.$$

A couple of characterizations for L_2-vanishment were presented in Zhou et al. (2011), based on which the following new design method, named L_2 low gain feedback, was introduced.

Definition 3 (L_2 *Low Gain Feedback*)**.** Assume that $(A, B) \in \left(\mathbf{R}^{n\times n}, \mathbf{R}^{n\times m}\right)$ is NCVE. A stabilizing feedback gain $K(\gamma) : [0, 1] \in \mathbf{R}^{m\times n}$ is said to be an L_2 low gain feedback if $(K(\gamma), A - BK(\gamma))$ is L_2-vanishing.

By using this L_2 low gain feedback, it is shown in Zhou et al. (2011) that Problem 1 can be solved for the special case of $\tau = 0$ under the condition of null controllability with vanishing energy. In this note, we will further show that Problem 1 is also solvable for the general situation of $\tau \neq 0$ under the same condition. Moreover, we can further impose, without loss of generality, the following assumption on the system (Zhou et al., 2011).

Assumption 1. $(A, B) \in \left(\mathbf{R}^{n\times n}, \mathbf{R}^{n\times m}\right)$ is controllable and all the eigenvalues of A are on the imaginary axis.

Consider the following family of linear systems:

$$\begin{cases} \dot{x}(t) = A(\gamma)\,x(t), & x(0) = x_0 \in \mathbf{R}^n, \\ y(t) = S(\gamma)\,x(t), \end{cases} \tag{3}$$

where $(S(\gamma), A(\gamma))$ is as defined in Definition 2 and $\gamma \in [0, 1]$. Notice that

$$\|y\|_{L_2} = \left(\int_0^\infty \left\| S(\gamma)\, e^{A(\gamma)t} x_0 \right\|^2 \mathrm{d}t \right)^{\frac{1}{2}}.$$

Then it follows from Definition 2 that $(S(\gamma), A(\gamma))$ is L_2-vanishing if and only if the L_2-norm of the output of system (3) with arbitrary bounded initial condition $x_0 \in \mathbf{R}^n$ approaches zero as γ does. This observation implies the possibility of extending the definition of L_2-vanishment for matrix pair $(S(\gamma), A(\gamma))$ to nonlinear systems.

Definition 4. Consider the family of nonlinear systems

$$\begin{cases} \dot{x}(t) = A(\gamma, x(t)), & x(0) = x_0 \in \mathbf{R}^n, \\ y(t) = S(\gamma, x(t)), \end{cases} \tag{4}$$

where $A(\gamma, x) : [0, 1] \times \mathbf{R}^n \to \mathbf{R}^n$ is continuous with respect to γ and globally Lipschitz with respect to x, $S(\gamma, x) : [0, 1] \times \mathbf{R}^n \to \mathbf{R}^m$ is continuous, and $\gamma \in [0, 1]$. Assume that, for an arbitrary $\gamma \in (0, 1]$, the system in (4) is globally asymptotically stable. Then the system in (4) is said to be L_2-vanishing if $\|x_0\| \le D < \infty \Rightarrow \lim_{\gamma\to 0^+} \|y\|_{L_2} = 0$.

The following simple results on L_2-vanishment can be easily derived. The idea found in the proof of this result will be adopted to prove our main results in the next section.