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Brief paper

Direct adaptive fuzzy control of nonlinear strict-feedback systems☆

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ABSTRACT

This paper focuses on adaptive fuzzy tracking control for a class of uncertain single-input /single-output nonlinear strict-feedback systems. Fuzzy logic systems are directly used to approximate unknown and desired control signals and a novel direct adaptive fuzzy tracking controller is constructed via backstepping. The proposed adaptive fuzzy controller guarantees that the output of the closed-loop system converges to a small neighborhood of the reference signal and all the signals in the closed-loop system remain bounded. A main advantage of the proposed controller is that it contains only one adaptive parameter that needs to be updated online. Finally, an example is used to show the effectiveness of the proposed approach.

1. Introduction

In the past years, backstepping-based nonlinear adaptive control has been paid considerable attention (Freeman & Kokotovic, 1996; Jiang & Hill, 1999; Kanellakopoulos, Kokotovic, & Morse, 1991; Krstic, Kanellakopoulos, & Kokotovic, 1992; Lin & Qian, 2001; Liu, Gu, & Zhou, 1999; Marino & Tomei, 1993a,b; Schwartz, Isidori, & Tarn, 1999). All the aforementioned works are based on the assumption that the uncertain nonlinearities in systems are either known functions whose parameters are unknown and linear with respect to those known functions, or bounded by known nonlinear functions. Thus, if such a prior knowledge of the structure or the upper-bounds of these unknown nonlinearities is not available, these approaches become infeasible. Such restrictions have been removed by using adaptive neural network control (Ge, Hang, & Zhang, 2000; Ge, Lee, & Harris, 1998; Ge & Wang, 2002; Kwan & Lewis, 2000; Lewis, Yesildirek, & Liu, 2000; Zhang, Ge, & Hang, 2000; Zhang, Peng, & Jiang, 2000), or adaptive fuzzy control (Wang & Mendel, 1992) (Chen, Li, & Chang, 1996; Tong & Li, 2003; Wang, 1993; Wang, Chan, & Liu, 2000). In the aforementioned literatures, neural networks or fuzzy logic systems are employed to approximate the unknown nonlinearities and the backstepping technique is implemented to construct controllers. The proposed adaptive controllers guarantee the uniform ultimate boundedness of all the signals in the closed-loop system. However, a common weakness of these control methods is that the number of adaptation laws depends on the number of the neural network nodes or the number of the fuzzy rule bases. With an increase of neural network nodes or fuzzy rules, the number of parameters to be estimated will increase significantly. As a result, the on-line learning time becomes prohibitively large. To solve this problem, Yang, Feng, and Ren (2004) and Yang, Zhou, and Ren (2003) considered the norm of the ideal weighting vector in fuzzy logic systems as the estimation parameter instead of the elements of weighting vector. Thus, the number of adaptation laws is reduced considerably.

Inspired by the work of Yang et al. (2003, 2004), we will develop a new direct adaptive fuzzy control approach in this paper. Unlike all the existing results on adaptive fuzzy control, Mamdani type fuzzy systems are used to directly approximate the desired control input signals instead of the unknown nonlinearities in systems. In this way, a direct adaptive fuzzy control method is developed. The main advantage of the developed method is that for an n-th order strict feedback nonlinear system, only one parameter is needed to be estimated on-line regardless of the number of fuzzy rule bases used. Therefore, the computation burden is significantly reduced and the algorithm is easily realized in practice.

2. Preliminaries and problem formulation

Consider the following SISO nonlinear system.

$$\dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t), \quad 1 \le i \le n-1,$$

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$$\dot{x}_n = f_n(\bar{x}) + g_n(\bar{x})u + d_n(t),$$
$$y = x_1, \qquad (1)$$

where $\bar{x} = [x_1, \dots, x_n]^T \in R^n$ is the state vector, $u \in R$ and $y \in R$ are the control input and output, respectively. $\bar{x}_i = [x_1, \dots, x_i]^T$, $f_i(.)$ and $g_i(.)$, $i = 1, 2, \dots, n$ are unknown nonlinear smooth functions with $f_i(0) = 0$, and $d_i(.)$, $i = 1, 2, \dots, n$, are the unknown external disturbances and satisfy $|d_i(t)| \le \bar{d}_i$ with $\bar{d}_i$ being a constant.

In this paper, a fuzzy logic system will be used to approximate a continuous function $f(x)$ defined on some compact set. Adopt the singleton fuzzifier, the product inference, and the center-average defuzzifier to deduce the following fuzzy rules:

R_i: IF x_1 is F_1^i and ... and x_n is F_n^i

THEN y is B^i $(i = 1, 2, \dots, N)$,

where $x = [x_1, \dots, x_n]^T \in R^n$ and $y \in R$ are the input and output of the fuzzy system, respectively, F_i^j and B^i are fuzzy sets in R. Since the strategy of singleton fuzzification, center-average defuzzification and product inference is used, the output of the fuzzy system can be formulated as

$$y(x) = \frac{\sum_{j=1}^{N} \bar{\Phi}_j \prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{N} [\prod_{i=1}^{n} \mu_{F_i^j}(x_i)]},$$

where $\bar{\Phi}_j$ is the point at which fuzzy membership function $\mu_{B^j}(\bar{\Phi}_j)$ achieves its maximum value, which is assumed to be 1. Let $p_j(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{N}[\prod_{i=1}^{n} \mu_{F_i^j}(x_i)]}$, $P(x) = [p_1(x), p_2(x), \dots, p_N(x)]^T$ and $\Phi = [\bar{\Phi}_1, \dots, \bar{\Phi}_N]^T$. Then the fuzzy logic system can be rewritten as

$$y(x) = \Phi^T P(x). \qquad (2)$$

If all memberships are chosen as Gaussian functions, the following lemma holds.

Lemma 1 (*Wang & Mendel, 1992*). *Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any given constant $\varepsilon > 0$, there exists a fuzzy logic system (2) such that*

$$\sup_{x \in \Omega} \left| f(x) - \Phi^T P(x) \right| \le \varepsilon.$$

Assumption 1. The sign of $g_i(\bar{x}_i)$ does not change and there exist constants b_m and b_M such that for $i = 1, \dots, n$,

$$0 < b_m \le |g_i(\bar{x}_i)| \le b_M.$$

Assumption 1 means that the unknown functions $g_i(.)$ are either strictly positive or negative. Without loss of generality, it is further assumed that $g_i \ge b_m$. In addition, because the constants b_m and b_M are not used for controller design, they can be unknown.

Assumption 2. The reference signal $y_d(t)$ and its time derivatives up to the n-th order are continuous and bounded.

3. Main result

For the system (1), backstepping-based design procedure contains n steps. At Step i $(1 \le i \le n)$, the desired but unknown control signal is first considered to stabilize the first i subsystems theoretically. Then, a fuzzy logic system $\Phi_i^T P_i(X_i)$ will be employed to approximate this unknown control signal, consequently, to construct the virtual control signal. The real tracking control law u will be designed at the last step. To develop a backstepping-based design procedure, we first define a constant as follows:

$$\theta = \max \left\{ \frac{1}{b_m} \|\Phi_i\|^2 : i = 1, 2, \dots, n \right\}. \qquad (3)$$

Obviously, θ is an unknown positive constant because b_m and $\|\Phi_i\|$ are unknown.

At Step i, the feasible virtual control signal is constructed as

$$\alpha_i(X_i) = \frac{-1}{2a_i^2} e_i \hat{\theta} P_i^T(X_i) P_i(X_i) \qquad (4)$$

where $e_i = x_i - \alpha_{i-1}$ with $\alpha_0 = y_d$, $X_i = [\bar{x}_i^T, \hat{\theta}, \bar{y}_d^{(i)T}]^T$, $\bar{y}_d^{(i)}$ denotes the vector of y_d and up to its i-th order time derivative. The virtual control signals $\hat{\alpha}_i$ will be specified at Step i.

Theorem 1. *For the reference signal $y_d(t)$, consider the system (1) satisfying Assumptions 1 and 2. Suppose that for $1 \le i \le n$, the packaged unknown functions $\hat{\alpha}_i$, $i = 1, 2, \dots, n$, can be approximated by the fuzzy logic systems in the sense that the approximating errors are bounded. If a control law is chosen as*

$$u = -\frac{1}{2a_n^2} e_n \hat{\theta} P_n^T(X_n) P_n(X_n), \qquad (5)$$

with the intermediate virtual control signals α_i defined by (4) and the adaptive law

$$\dot{\hat{\theta}} = \sum_{i=1}^{n} \frac{r}{2a_i^2} (x_i - \alpha_{i-1})^2 P_i^T(X_i) P_i(X_i) - k_0 \hat{\theta}, \qquad (6)$$

where the constants $r > 0$, $k_0 > 0$, and $a_i > 0$ $(i = 1, 2, \dots, n)$ are design parameters, then all the signals in the closed-loop system remain bounded. Furthermore, for any given scalar $\varepsilon > 0$, the controller parameters can be tuned such that

$$\lim_{t \to \infty} \|y - y_d\|^2 \le \varepsilon^2,$$

The proof of Theorem 1 consists of two steps. First, a systematic design scheme is presented based on the backstepping approach. Then, the stability analysis of the closed-loop system is carried out.

3.1. Adaptive fuzzy control design

In the following, for the purpose of simplicity, the time variable t and the state vector $\bar{x}_i$ will be omitted from function expressions and let $P_i = P_i(X_i)$. In addition, it is easy to prove from (6) that if $\hat{\theta}(0) \ge 0$, then $\hat{\theta}(t) \ge 0$ for all $t \ge 0$. In fact, it is always reasonable to choose $\hat{\theta}(0) \ge 0$ in a practical situation, as $\hat{\theta}$ is an estimation of θ. This conclusion will be used in each design step.

Step 1. Define the tracking error as $e_1 = x_1 - y_d$ and consider a Lyapunov function candidate as

$$V_1 = \frac{1}{2} e_1^2 + \frac{b_m}{2r} \tilde{\theta}^2, \qquad (7)$$

where $\tilde{\theta} = \theta - \hat{\theta}$. Differentiating V_1 yields

$$\dot{V}_1 = e_1 (f_1 + g_1 x_2 - \dot{y}_d + d_1) - \frac{b_m}{r} \tilde{\theta} \dot{\hat{\theta}}$$
$$\le e_1 \left(g_1 x_2 + \bar{f}_1 \right) + \frac{1}{2} \rho^2 \bar{d}_1^2 - \frac{1}{2} g_1^{-2} e_1^2 - \frac{b_m}{r} \tilde{\theta} \dot{\hat{\theta}}, \qquad (8)$$

where $\bar{f}_1(X_1) = f_1 + \frac{1}{2}\rho^{-2} e_1 + \frac{1}{2} g_1^{-2} e_1 - \dot{y}_d$ and the completion of squares are used for the term $e_1 d_1$ with $\rho > 0$ being a constant. To stabilize this system, take the intermediate control signal as $\hat{\alpha}_1(X_1) = -g_1^{-1} \left\{ k_1 e_1 + \bar{f}_1 \right\}$ with k_1 being a positive constant. Further, add and subtract $g_1 \hat{\alpha}_1$ in the last bracket in (8) to obtain the following inequality.

$$\dot{V}_1 \le -k_1 e_1^2 + e_1 g_1 \left(x_2 - \hat{\alpha}_1 \right)$$
$$+ \frac{1}{2} \rho^2 \bar{d}_1^2 - \frac{1}{2} g_1^{-2} e_1^2 - \frac{b_m}{r} \tilde{\theta} \dot{\hat{\theta}}. \qquad (9)$$

However, $\hat{\alpha}_1$ cannot be implemented in practice as it contains the unknown functions f_1 and g_1. Thus, according to Lemma 1, for any given $\varepsilon_1 > 0$, there exists a fuzzy logic system $\Phi_1^T P_1(X_1)$ such that