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A convex characterization of dynamically-constrained anti-windup controllers ☆

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ABSTRACT

Based on a recent description of deadzone nonlinearities via local sector conditions, a new LMI characterization is proposed to compute full-order continuous-time anti-windup controllers with pole constraints. More precisely, an upper bound is introduced on the real part of the controller poles to avoid slow dynamics, which often leads to poor time-domain performance. As is demonstrated in a short applicative part, the introduction of such a bound allows us to efficiently handle the trade-off between stability domain enlargement and time-domain response relevance.

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1. Introduction

As is observed in Grimm et al. (2003), one of the most challenging problems in nonlinear control is certainly to design high performance control laws for linear systems with input saturations. Numerous methods now exist to handle such nonlinearities, but the most popular and pragmatic one remains the anti-windup approach, which has received much attention in the past decade. Let us notably cite Kothare, Campo, Morari, and Nett (1994), where a unifying framework inspired by the famous standard forms from robust control theory is developed. More recently, based on the LFT/LPV framework, extended anti-windup schemes were introduced in Lu, Wu, and Kim (2005), Saeki and Wada (2002), Turner and Postlethwaite (2004) and Wu and Soto (2004). In these contributions, the saturations are viewed as sector nonlinearities and the anti-windup control design issue is recast into a convex optimization problem under LMI constraints. Following a similar path, alternative techniques using a less conservative representation of the saturation function based on a modified sector condition (Gomes da Silva Jr. & Tarbouriech, 2005) were then proposed to compute either static (Biannic, Tarbouriech, & Farret, 2006; Gomes da Silva Jr. & Tarbouriech, 2005) or dynamic (Hu, Teel, & Zaccarian, 2005; Kiyama & Sawada, 2004; Tarbouriech, Gomes da Silva, & Bender, 2006) anti-windup controllers. These techniques are further exploited throughout this note to compute continuous-time dynamic anti-windup controllers, which offer more flexibility compared to static gains. Interestingly, the problem is shown to be convex when the order of the anti-windup compensator coincides with that of the nominal closed-loop plant. However, from a practical point of view, it is usually observed that such full-order controllers exhibit slow dynamics, which remains visible on the plant outputs even when the saturations are no longer active. To cope with this problem and improve the time-domain performance of saturated plants, a new convex characterization is proposed in this paper to compute dynamically-constrained anti-windup compensators. More precisely, an upper bound is introduced on the real part of their poles.

The note is organized as follows. Section 2 introduces some useful notation and backgrounds. The main technical result is then stated and proved in Section 3. A short application is finally proposed in Section 4, which clearly illustrates the interest of constraining the poles of the anti-windup controller.

2. Notation and backgrounds

The considered anti-windup design problem is illustrated in Fig. 1. $M(s)$ is a strictly proper linear plant in feedback-loop with normalized deadzone nonlinearities Φ. Note that $\Phi(z) = z - \Psi(z)$, where Ψ denotes the standard saturation operator. It can be assumed without loss of generality that $M(s)$ is stable, since it is composed of both the plant to be controlled (including actuators)

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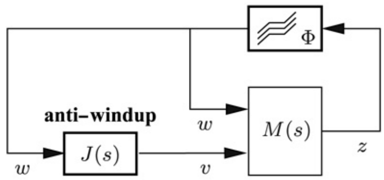

Fig. 1. A standard form for anti-windup design.

and the nominal controller. Let this closed-loop plant be described by:

$$M(s):\begin{cases}\dot{\xi}=A\xi+B_\phi w+B_a v, & \xi\in\mathbb{R}^{n_M}\\ z=C_\phi\xi\in\mathbb{R}^m.\end{cases} \quad (1)$$

The aim of this paper is to compute a dynamic anti-windup compensator $J(s)=C_J(sI_{n_J}-A_J)^{-1}B_J+D_J$ to enlarge the stability domain of the nonlinear interconnection depicted on Fig. 1, whose equations are obtained as follows (with $n=n_M+n_J$):

$$\begin{cases}\dot{x}=\begin{bmatrix}A & B_aC_J\\ 0 & A_J\end{bmatrix}x+\begin{bmatrix}B_\phi+B_aD_J\\ B_J\end{bmatrix}w\\ z=\begin{bmatrix}C_\phi & 0\end{bmatrix}x, \quad x\in\mathbb{R}^n\\ w=\Phi(z).\end{cases} \quad (2)$$

The main result stated in Section 3 is based on the following theorem, which is a straightforward adaptation of Biannic et al. (2006) and Gomes da Silva Jr. and Tarbouriech (2005). Note that for compactness the symmetric terms in the matrix inequalities are replaced by ⋆ in the remaining of the paper.

Theorem 2.1 (*Dynamic Anti-Windup Synthesis*). *Let a positive scalar ρ and a polyhedral set $\overline{\mathcal{X}}=\mathrm{co}(\{\overline{\chi}_1,\ldots,\overline{\chi}_q\})\subset\mathbb{R}^n$ where co(.) denotes the convex hull and $\overline{\chi}_i^{\mathrm{T}}=[\chi_i^{\mathrm{T}}\ 0_{n_M}]$ be given. If there exist a symmetric matrix $Q\in\mathbb{R}^{n\times n}$, a diagonal matrix $S\in\mathbb{R}^{m\times m}$, a full rectangular matrix $Z\in\mathbb{R}^{m\times n}$ and matrices A_J, B_J, C_J, D_J of appropriate dimensions such that the following LMI conditions hold:*

$$\begin{pmatrix}Q & \star\\ \rho\overline{\chi}_i^{\mathrm{T}} & 1\end{pmatrix}>0,\quad i=1\ldots q \quad (3)$$

$$\begin{pmatrix}\begin{bmatrix}A & B_aC_J\\ 0 & A_J\end{bmatrix}Q & 0\\ S\begin{bmatrix}B_\phi+B_aD_J\\ B_J\end{bmatrix}^{\mathrm{T}}-Z & -S\end{pmatrix}+(\star)<0 \quad (4)$$

$$\begin{pmatrix}Q & \star\\ Z_i+[C_{\phi_i}\ \ 0]Q & 1\end{pmatrix}>0,\quad i=1\ldots m \quad (5)$$

where Z_i and C_{ϕ_i} denote the ith rows of Z and C_ϕ respectively, then the ellipsoid:

$$\mathcal{E}_P=\{x\in\mathbb{R}^n, x^{\mathrm{T}}Px\le 1\}\supset\rho\overline{\mathcal{X}} \quad (6)$$

where $P=Q^{-1}$, defines a domain of asymptotic stability for the nonlinear interconnection (2).

Note that the order of the anti-windup controller is free, but the problem is not convex unless A_J and C_J are fixed.

3. Main result

With the notation of Theorem 2.1 in mind, the main result of the paper can now be stated.

Theorem 3.1 (*Full-order Anti-Windup Synthesis with Pole Constraints*). *Let two positive scalars λ, ρ and a polyhedral set $\overline{\mathcal{X}}$ as defined above be given. If there exist symmetric matrices $X, Y\in\mathbb{R}^{n_M\times n_M}$, a diagonal matrix $S\in\mathbb{R}^{m\times m}$ and a full rectangular matrix $W=[U\ V]\in\mathbb{R}^{m\times(n_M+n_M)}$ such that the following LMI conditions hold:*

$$X>\rho^2\chi_i\chi_i^{\mathrm{T}},\quad i=1\ldots q \quad (7)$$

$$\begin{pmatrix}N_a^{\mathrm{T}}(AY+YA^{\mathrm{T}})N_a & \star\\ (SB_\phi^{\mathrm{T}}-V)N_a & -2S\end{pmatrix}<0 \quad (8)$$

$$\begin{pmatrix}AX+XA^{\mathrm{T}}-2\lambda X & \star\\ 2\lambda Y & -2\lambda Y\end{pmatrix}<0 \quad (9)$$

$$\begin{pmatrix}X & \star & \star\\ X & Y & \star\\ U_i & V_i+C_{\phi_i}Y & 1\end{pmatrix}>0,\quad i=1\ldots m \quad (10)$$

where N_a denotes any basis of the null-space of B_a^{T}, while U_i, V_i and C_{ϕ_i} are the ith rows of U, V and C_ϕ respectively, then there exist a dynamic anti-windup controller $J(s)$ whose poles $\lambda_1,\lambda_2,\ldots,\lambda_{n_M}$ satisfy:

$$\Re(\lambda_j)<-\lambda,\quad j=1\ldots n_M \quad (11)$$

and a positive definite matrix $P\in\mathbb{R}^{n\times n}$ such that the ellipsoid:

$$\mathcal{E}_P=\left\{x\in\mathbb{R}^n, x^{\mathrm{T}}Px\le 1\right\}\supset\rho\overline{\mathcal{X}} \quad (12)$$

defines a domain of asymptotic stability for the nonlinear interconnection (2).

Proof. The poles λ_j of the anti-windup controller $J(s)$ satisfy $\Re(\lambda_j)<-\lambda$ *iff* there exists a positive definite matrix $H\in\mathbb{R}^{n_M\times n_M}$ such that:

$$A_JH+HA_J^{\mathrm{T}}+2\lambda H<0. \quad (13)$$

With reference to Theorem 2.1, let us first partition the matrices Q and $P=Q^{-1}$ as[1]:

$$Q=\begin{pmatrix}Y & N^{\mathrm{T}}\\ N & F\end{pmatrix},\qquad P=\begin{pmatrix}X^{-1} & M^{\mathrm{T}}\\ M & E\end{pmatrix} \quad (15)$$

where $X, Y\in\mathbb{R}^{n_M\times n_M}$. Choosing $H=F$, it is readily checked that the inequality:

$$\begin{pmatrix}\begin{bmatrix}A & B_aC_J\\ 0 & A_J\end{bmatrix}Q+\begin{bmatrix}0 & 0\\ 0 & \lambda F\end{bmatrix} & 0\\ S\begin{bmatrix}B_\phi+B_aD_J\\ B_J\end{bmatrix}^{\mathrm{T}}-Z & -S\end{pmatrix}+(\star)\quad<0 \quad (16)$$

enforces both (4) and (13). Then, gathering the anti-windup state matrices in $\Omega=\begin{pmatrix}A_J & B_J\\ C_J & D_J\end{pmatrix}$ and partitioning $Z=(V\ \ \tilde{U})$, where $V,\tilde{U}\in\mathbb{R}^{m\times n_M}$, inequality (16) becomes:

$$\Theta+\mathcal{U}^{\mathrm{T}}\Omega\mathcal{V}+\mathcal{V}^{\mathrm{T}}\Omega^{\mathrm{T}}\mathcal{U}<0 \quad (17)$$

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[1] Inequality (10) implies $\begin{pmatrix}X & X\\ X & Y\end{pmatrix}>0$, *i.e.* $X^{-1}>0$, $Y>0$ and $X^{-1}Y>I$, which means that the conditions of the completion lemma (as stated in Packard, Zhou, Pandey, and Becker (1991)) are strictly verified here. As a result, the partition (15) is valid. Moreover, following Gahinet and Apkarian (1994), the matrix Q can be obtained as:

$$Q=\begin{pmatrix}Y & I\\ N & 0\end{pmatrix}\begin{pmatrix}I & X^{-1}\\ 0 & M\end{pmatrix}^{-1} \quad (14)$$

where the nonsingular square matrices M and N are the solutions of $M^{\mathrm{T}}N=I-X^{-1}Y<0$.

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