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Long-term fading channel estimation from sample covariances[☆]

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ABSTRACT

The power attenuation dynamics for long-term log-normal fading channels in wireless communication systems is modeled by a stochastic differential equation. Estimators for the model parameters based on sample covariances from data corrupted by discrete-time measurement noise are given. It is shown that the estimators are consistent, and variance expressions are derived and compared numerically to the Cramér–Rao bound. The estimation of the model parameters allows for the design of power control algorithms.

1. Introduction

In wireless communication systems, long-term (or large-scale) fading is caused by shadowing effects due to buildings and moving obstacles. The power attenuation dynamics for long-term log-normal fading channels can be described by the stochastic differential equation

$$\mathrm{d}x(t) = -a_0(x(t) + b_0)\mathrm{d}t + \mathrm{d}w(t), \tag{1}$$

see Charalambous and Menemenlis (1999), Huang, Caines, and Malhamé (2004) and Mossberg, Larsson and Mossberg (2006c). The equation is a mean reverting Ornstein–Uhlenbeck process (Kloeden & Platen, 1992), where $x(t)$ denotes the attenuation at time t of the power of a mobile user in dB, $a_0 > 0$ is the speed of a long-term adjustment of $x(t)$ towards the mean $-b_0$, and $\mathrm{d}w(t)$ is the increment of a Wiener process $w(t)$. The increment $dw(t)$ has variance σ_0^2 and the Wiener process $w(t)$ is independent of the Gaussian random variable $x(0)$. Furthermore, $x(0)$ is assumed to have mean $-b_0$ and intensity $\sigma_0^2/(2a_0)$. The actual signal attenuation is given by $\mathrm{e}^{x(t)}$ and the received signal $\bar{s}(t)$ of the mobile at the base station is given by

$$\bar{s}(t) = \sqrt{p(t)}\mathrm{e}^{cx(t)}s(t), \tag{2}$$

where $p(t)$ is the transmitted power of the mobile, $s(t)$ is the signal sent by the mobile, and $c = -\ln(10)/20$. This paper deals with the problem of finding estimators of the model parameters from irregularly sampled data corrupted by measurement noise. One important motivation for this is the design of power control algorithms, see, e.g., Huang et al. (2004) and Olama, Djouadi, and Charalambous (2007) for further details. See also Charalambous and Menemenlis (1999, 2001) for further examples of modeling in communications using the mean reverting Ornstein–Uhlenbeck process. Description and identification of stochastic systems is a well-established subject area, see, e.g., Caines (1988), Ljung (1999) and Söderström and Stoica (1989) and the references therein, with applications in many different fields. This paper is an example of a work on continuous-time stochastic systems, a topic that is treated in the recent book (Garnier & Wang, 2008).

It is assumed that the irregularly sampled data

$$\boldsymbol{\psi} = \begin{bmatrix} \psi(t_1) & \cdots & \psi(t_{\tilde{N}}) \end{bmatrix}^{\mathrm{T}} \tag{3}$$

are available, where

$$\psi(t_k) = x(t_k) + e(t_k), \tag{4}$$

with $e(t_k)$ being zero mean discrete-time Gaussian white noise of variance λ_0^2. Fast and accurate estimators, robust to the

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measurement noise $e(t_k)$, of the parameters

$$\boldsymbol{\theta}_0 = \begin{bmatrix} a_0 & b_0 & \sigma_0^2 & \lambda_0^2 \end{bmatrix}^{\mathrm{T}} \tag{5}$$

from the irregularly sampled data $\boldsymbol{\psi}$ are presented in the paper. The estimators, except the one for b_0, are based on sample covariances. A preliminary version (without any analysis) of this paper can be found in Mossberg, Larsson and Mossberg (2006b). Examples of other estimators for long-term fading channels found in the literature are the least squares based estimators in Mossberg et al. (2006c), the instrumental variable based estimators in Mossberg, Larsson and Mossberg (2006a), and the estimators based on the expectation maximization algorithm in Olama, Jaladhi, Djouadi, and Charalambous (2007).

In order to get a well-posed problem, we make the following assumptions and introduce the following notations.

1. In what follows, $h_k = t_{k+1} - t_k$, $0 < \underline{h} < h_k \leqslant \bar{h}$, for $\bar{h}$ sufficiently bounded, and let $\gamma_k(\mu) = t_{k+\mu} - t_k$.
2. The sequence of sampling intervals, $\{h_k\}$, is independent and identically distributed (i.i.d.) with an associated probability density function $p(h_k)$. Moreover, $\{h_k\}$ is independent of the process $x(t)$.
3. The notation $\mathrm{E}\{\cdot\}$ means expectation with respect to the process, given a certain sampling scheme with h_k bounded as described above.
4. Furthermore, $\mathrm{E}_h\{\cdot\}$ means expectation with respect to the sampling.
5. The notation $\mathrm{E}_{p,h}\{\cdot\}$ means expectation with respect to both the process and the sampling.
6. Throughout the paper, for ease of notation, N will always denote the largest possible value of the sample index that can be used for a given case.

The outline of the paper is as follows. The proposed estimators are described in Section 2. Consistency of the estimators is shown in Section 3, and a variance analysis is carried out in Section 4. A numerical study is presented in Section 5, and conclusions are drawn in Section 6.

2. The estimators

In this section, estimators of the parameters in $\boldsymbol{\theta}_0$ are proposed. We start by estimating b_0 which is done straightforwardly as follows.

Proposition 1. *Since $\mathrm{E}\{x(t)\} = -b_0$, a natural estimator of b_0 is given as*

$$\hat{b} = -\frac{1}{N}\sum_{k=1}^{N}\psi(t_k). \tag{6}$$

In order to find estimators for a_0, σ_0^2, and λ_0^2, we first note that the system (1) can be rewritten as

$$\mathrm{d}y(t) = -a_0 y(t)\mathrm{d}t + \mathrm{d}w(t), \tag{7}$$

$$x(t) = y(t) - b_0. \tag{8}$$

It is readily verified, for example by solving the Yule–Walker equations, that $y(t)$ in (7) has the covariance function

$$r_y(\tau) = \mathrm{E}\{y(t+\tau)y(t)\} = \frac{\sigma_0^2}{2a_0}e^{-a_0|\tau|}, \tag{9}$$

since $\mathrm{E}\{y(t)\} = 0$. Then, since $\mathrm{E}\{x(t)\} = -b_0$, it follows from (8) that

$$r_x(\tau) = \mathrm{E}\{(x(t+\tau)+b_0)(x(t)+b_0)\} = r_y(\tau), \tag{10}$$

so

$$\begin{aligned} r_x\big(\gamma_k(\mu)\big) &= \mathrm{E}\{(x(t_{k+\mu})+b_0)(x(t_k)+b_0)\} \\ &= r_y\big(\gamma_k(\mu)\big). \end{aligned} \tag{11}$$

Furthermore,

$$\begin{aligned} r_\psi\big(\gamma_k(\mu)\big) &= \mathrm{E}\{(\psi(t_{k+\mu})+b_0)(\psi(t_k)+b_0)\} \\ &= \mathrm{E}\{(x(t_{k+\mu})+e(t_{k+\mu})+b_0)(x(t_k)+e(t_k)+b_0)\} \\ &= r_x\big(\gamma_k(\mu)\big) + \lambda_0^2\delta_{\mu,0}, \end{aligned} \tag{12}$$

where $\delta_{\mu,0}$ is the Kronecker delta.

Introduce

$$\hat{r}_\psi(\mu) = \frac{1}{N}\sum_{k=1}^{N}\{(\psi(t_{k+\mu})+\hat{b})(\psi(t_k)+\hat{b})\}. \tag{13}$$

Moreover, let

$$\hat{\mathbf{r}} = \begin{bmatrix} \hat{r}_\psi(1) & \cdots & \hat{r}_\psi(m) \end{bmatrix}^{\mathrm{T}}, \tag{14}$$

$$\mathbf{f}_k(a) = \begin{bmatrix} \mathrm{e}^{-a\gamma_k(1)} & \cdots & \mathrm{e}^{-a\gamma_k(m)} \end{bmatrix}^{\mathrm{T}}, \tag{15}$$

$$\bar{\mathbf{f}}(a) = \frac{1}{N}\sum_{k=1}^{N}\mathbf{f}_k(a), \tag{16}$$

$$\boldsymbol{\ell}(a) = \frac{\bar{\mathbf{f}}(a)}{2a}, \tag{17}$$

where $m > 1$. Define

$$\boldsymbol{\vartheta} = \begin{bmatrix} a & \sigma^2 \end{bmatrix} \tag{18}$$

and

$$V(\boldsymbol{\vartheta}) = \|\hat{\mathbf{r}} - \boldsymbol{\ell}(a)\sigma^2\|_2^2 \tag{19}$$

from which an estimate $\hat{\boldsymbol{\vartheta}}$ of

$$\boldsymbol{\vartheta}_0 = \begin{bmatrix} a_0 & \sigma_0^2 \end{bmatrix} \tag{20}$$

is given as

$$\hat{\boldsymbol{\vartheta}} = \arg\min_{\boldsymbol{\vartheta}} V(\boldsymbol{\vartheta}). \tag{21}$$

From (19), the estimate

$$\hat{\sigma}^2(a) = \big(\boldsymbol{\ell}^{\mathrm{T}}(a)\boldsymbol{\ell}(a)\big)^{-1}\boldsymbol{\ell}^{\mathrm{T}}(a)\hat{\mathbf{r}} \tag{22}$$

is given as a function of a.

Proposition 2. *By inserting (22) in (19), an estimate of a_0 is obtained as*

$$\begin{aligned} \hat{a} &= \arg\min_a \|\hat{\mathbf{r}} - \boldsymbol{\ell}(a)\hat{\sigma}^2(a)\|_2^2 \\ &= \arg\min_a \|\big(\mathbf{I} - \mathbf{P}(a)\big)\hat{\mathbf{r}}\|_2^2 \\ &= \arg\max_a W(a), \end{aligned} \tag{23}$$

where

$$\mathbf{P}(a) = \boldsymbol{\ell}(a)\big(\boldsymbol{\ell}^{\mathrm{T}}(a)\boldsymbol{\ell}(a)\big)^{-1}\boldsymbol{\ell}^{\mathrm{T}}(a), \tag{24}$$

$$W(a) = \|\mathbf{P}(a)\hat{\mathbf{r}}\|_2^2, \tag{25}$$

where the projection matrix $\mathbf{P}(a)$ has the properties $\mathbf{P}(a) = \mathbf{P}^{\mathrm{T}}(a)$ and $\mathbf{P}(a) = \mathbf{P}^2(a)$.

Proposition 3. *An estimate of σ_0^2 is found by inserting (23) in (22).*