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# Peaking free variable structure control of uncertain linear systems based on a high-gain observer☆

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ABSTRACT

An output-feedback model-reference variable structure controller based on a high-gain observer (HGO) is proposed and analyzed. For single-input–single-output (SISO) linear plants with relative degree greater than one, the control law is generated using the HGO signals only to drive the *sign* function of the variable structure control component while the *sign* function gain, also called *modulation*, as well as the other components of the control signal are generated using signals from state variable filters which do not require high gain and are free of peaking. This scheme achieves global exponential stability with respect to a small residual set and does not generate peaking in the control signal.



## 1. Introduction

This paper is concerned with the design of output feedback control laws for uncertain linear systems using variable structure control (VSC) and a high-gain observer (HGO). Here, the generation of large fast transients in the control signal, known as *peaking phenomena*, is avoided while preserving global stability.

VSC is an efficient tool to design controllers for plants under significant uncertainty conditions. Owing to the practical difficulty of measuring all states, as required in early works, output-feedback strategies for VSC were proposed (e.g. Emelyanov, Korovin, Nersisian, and Nisenzon (1992), Esfandiari and Khalil (1992) and Walcott and Żak (1988)). Recently, higher order sliding modes for plants of arbitrary relative degree have been also considered by Levant (1998) using robust exact differentiators. Theoretically, controllers based on such differentiators may lead to exact output tracking. However, stability and/or convergence of the overall control system was guaranteed only locally.

While it is well known from the seminal paper (Bondarev, Bondarev, Kostyleva, & Utkin, 1985) that output feedback sliding mode control is possible with the use of asymptotic observers, a good knowledge of the plant model is needed. For uncertain plants, a solution for state estimation is the HGO, which is robust to model uncertainties (Emelyanov et al., 1992; Esfandiari & Khalil, 1992). A drawback of the HGO is the *peaking phenomenon*, which may be destabilizing and even provoke finite-time escape in closed-loop nonlinear systems (Atassi & Khalil, 2000; Sussmann & Kokotović, 1991).

The relevance of peaking elimination is explained as follows. If the system satisfies a global Lipschitz condition (e.g., if the system is linear), global asymptotic stability can be obtained with HGOs (Busawon, El Assoudi, & Hammouri, 1993; Gauthier, Hammouri, & Othman, 1992), at the expense of unacceptable transient responses (Atassi & Khalil, 2000). In linear plants with actuator constraints, peaking can lead to saturation of the control signal and, consequently, to performance degradation, as pointed out by Méndez-Acosta, Femat, and Campos-Delgado (2004). Moreover, peaking may cause undesirable mechanical wear and energy loss. For example, in flow control systems operating at high flow rates, large peaks in the control signal can abruptly close a valve, causing water hammer which can severely damage pipes, valves and other mechanical parts. These facts are well known in industrial process control when derivative action is used.

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Consequently, peaking avoidance has motivated the development of several control design remedies (Li, Ang, & Chong, 2006).

Several previous works give some alternatives for peaking alleviation, for instance: (1) The amplitude of the control signal can be globally bounded through saturation (Oh & Khalil, 1995, 1997). This may restrict stability to become local or semi-global and precludes global stabilization of unstable linear systems; (2) The HGO *free of peaking* proposed by Chitour (2002) is based on a time-varying observer gain. However, this algorithm may fail in actual systems since disturbances may excite peaking when the observer gain is large. (3) The *semi-high gain observer* (Lu & Spurgeon, 1998), which is a HGO with a non-conservative value for the observer gain, is computed such that closed-loop stability is guaranteed. However, this procedure, which was developed for stabilization purposes, seems inadequate for tracking applications where the observer gain must be large enough to keep the residual output error small.

This paper presents an output-feedback model-reference variable structure controller that uses simultaneously a HGO and state variable filters to implement the control law. The main contributions are: (i) To introduce a HGO-based controller for uncertain linear plants which is free of peaking and yet guarantees global exponential stability with respect to a small residual set. To the best of our knowledge, such a result is original. The importance of the absence of peaking is illustrated by experimental results in Section 6.2. (ii) To offer a significantly more simple controller than the earlier variable structure model-reference robust controller (VS-MRRC)[1] (Hsu, Araújo, & Costa, 1994; Hsu, Lizarralde, & Araújo, 1997). The VS-MRRC also solves the output-feedback tracking control problem, however the number of required state variables significantly exceeds the HGO by $2(n+1)(n^*-1)-1$, where $n$ is the order and $n^*$ is the relative degree of the plant. For example, if $n=5$ and $n^*=3$ the HGO-based controller requires 14 state variables, while the VS-MRRC requires 37 state variables. This is due to the more involved structure of the VS-MRRC which requires several filters for the computation of the gains of $n^*$ modulated relays, instead of a single modulated relay in the new HGO-based controller. (iii) The HGO structure allows a more natural extension to nonlinear plants with nonlinearities depending on unmeasured states (e.g. Oliveira, Peixoto, & Hsu, 2008). Such an extension is not trivial for the VS-MRRC.

*Notations:* The $\mathcal{L}_{\infty e}$ norm of the signal $x(t) \in \mathbb{R}^n$ is defined as $\|x_{t,t_0}\|_\infty := \sup_{t_0 \le \tau \le t} \|x(\tau)\|$. A mixed time-domain and Laplace transform domain is adopted, i. e., $s$ denotes either the Laplace variable or the differential operator, according to the context. The output signal $y$ of a linear time-invariant system with transfer function matrix $H(s)$ and input $u$ is denoted by $H(s)u$. The time-domain convolution is denoted by $h(t) * u(t)$.

## 2. Problem statement and preliminaries

Let a linear, time-invariant, observable and controllable plant be described by the input–output model

$$y = G(s)[u + d(t)], \qquad G(s) = k_p \frac{N_p(s)}{D_p(s)}, \tag{1}$$

where $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output, $d \in \mathbb{R}$ is an unmeasured input disturbance, $k_p \in \mathbb{R}$ is the high frequency gain, $N_p(s)$ and $D_p(s)$ are monic polynomials. The following assumptions regarding the plant are usual in model-reference adaptive control (Ioannou & Sun, 1996): (A1) $G(s)$ is minimum phase; (A2) $G(s)$ is strictly proper; (A3) the order of the system ($n$) is known; (A4) the relative degree of $G(s)$, $n^*$, is known; (A5) the sign of $k_p$ is known and $k_p > 0$ for simplicity.

Two additional assumptions are needed in the design of the modulation function of the control law: (A6) the disturbance $d(t)$ is piecewise continuous and a bound $\bar{d}(t)$ is known such that $|d(t)| \le \bar{d}(t) \le \bar{d}_{\sup} < +\infty$, $\forall t \ge 0$; (A7) the parameters of $G(s)$ are uncertain but the coefficients of $D_p(s)$ and $N_p(s)$ belong to known bounded sets and, a bound $\underline{k}_p$ is known such that $0 < \underline{k}_p \le k_p$.

To simplify the analysis and design of the controller, *the reference model* is defined by

$$y_M = W_M(s) r, \qquad W_M(s) = \frac{k_M}{L(s)\,(s+\gamma)}, \tag{2}$$

where $y_M(t)$ is the output signal, $r(t)$ is a piecewise continuous and uniformly bounded reference signal, $k_M > 0$ is the high frequency gain of the model, $L(s)$ is a monic Hurwitz polynomial of degree $N := n^* - 1$ and $\gamma > 0$.

*The objective* is to design an output-feedback controller to achieve asymptotic convergence of the output error

$$e(t) := y(t) - y_M(t) \tag{3}$$

to zero, or to some small residual neighborhood of zero.

If the plant and the disturbance $d(t)$ are perfectly known, a control law which achieves matching between the closed-loop transfer function and $W_M(s)$ is given by (Cunha, Hsu, Costa, & Lizarralde, 2003)

$$u^* = \theta^{*T}\omega - w_d(t) * d(t), \tag{4}$$

where $w_d(t)$ is the impulse response of a system with transfer function $W_d(s) = 1 - \theta_1^{*T} A(s)/\Lambda(s)$, where $A(s) = [s^{n-2}, s^{n-3}, \ldots, s, 1]^{\mathrm{T}}$, and $\Lambda(s)$ is an arbitrary monic Hurwitz polynomial of degree $n-1$. The signal $w_d(t) * d(t)$ cancels the input disturbance $d(t)$. The parameter vector is given by $\theta^{*T} = \left[\theta_1^{*T}, \theta_2^{*T}, \theta_3^*, \theta_4^*\right]$, with $\theta_1^*, \theta_2^* \in \mathbb{R}^{(n-1)}$, $\theta_3^*, \theta_4^* \in \mathbb{R}$ and the regressor vector is $\omega = \left[\omega_1^{\mathrm{T}}, \omega_2^{\mathrm{T}}, y, r\right]^{\mathrm{T}}$ with state variable filters:

$$\omega_1 = \frac{A(s)}{\Lambda(s)} u, \qquad \omega_2 = \frac{A(s)}{\Lambda(s)} y. \tag{5}$$

The matching parameters $\theta^*$ can be computed from $\theta_4^* = k_p^{-1} k_M$ and a Diophantine equation (Ioannou & Sun, 1996, eq. (6.3.13)).

Consider the system state $X := [x_p^{\mathrm{T}}, \omega_1^{\mathrm{T}}, \omega_2^{\mathrm{T}}]^{\mathrm{T}}$, where $x_p \in \mathbb{R}^n$ is the plant state, and a non-minimal realization $\{A_c, B_c, C_o\}$ of $W_M(s)$ with state vector $X_M$ and $A_c$ Hurwitz. Then, the state error $X_e := X - X_M$ and the output error $e(t)$ satisfy

$$\dot{X}_e = A_c X_e + B_c k \left[u - \theta^{*T}\omega + w_d(t) * d(t)\right], \tag{6}$$

$$e = C_o X_e, \tag{7}$$

where $k := (\theta_4^*)^{-1} = k_M^{-1} k_p$ (Hsu et al., 1994). For the HGO design and overall stability analysis, a reduced order error model is advantageous. To this end, consider a Kalman decomposition (Kailath, 1980, pp. 132–134) for the system (6) and (7) with partial observable states $x_{oc}$ (controllable) and $x_{o\bar{c}}$ (uncontrollable) satisfying:

$$\dot{x}_{oc} = A_{11} x_{oc} + A_{12} x_{o\bar{c}} + B_1 k \left[u - u^*\right], \tag{8}$$

$$\dot{x}_{o\bar{c}} = A_{22} x_{o\bar{c}}, \tag{9}$$

$$e = C_1 x_{oc} + C_2 x_{o\bar{c}}, \tag{10}$$

where $\{A_{11}, B_1, C_1\}$ is a minimal realization of $W_M(s)$. The characteristic polynomial of $A_{11}$ is $D_M(s) = L(s)(s+\gamma) = s^{n^*} + a_{n^*-1} s^{n^*-1} + \cdots + a_1 s + a_0$. Noting that $C_1 A_{11}^{i-1} B_1 = 0$

[1] Originally, it was named *variable structure model-reference adaptive controller* (VS-MRAC) due to its close relation with MRAC.