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Global target aggregation and state agreement of nonlinear multi-agent systems with switching topologies ☆

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ABSTRACT

In this paper, we discuss coordination problems of a group of autonomous agents, including the target aggregation to a convex set and the state agreement. The aggregation of the whole agent group, consisting of leaders (informed agents) and followers, to a given set is investigated with switching interconnection topologies described by the connectivity assumptions on the joint topology in the time interval $[t, +\infty)$ for any time t, and then the state agreement problem is studied in a similar way. An approach based on set stability and limit set analysis is given to study the multi-agent convergence problems. With the help of graph theory and convex analysis, coordination conditions are obtained in some important cases, and the results show that simple local rules can make the networked agents with first-order nonlinear individual dynamics achieve desired collective behaviors.

1. Introduction

Recent years have seen a large and growing literature concerned with the coordination of a group of autonomous agents, partly due to a broad application of multi-agent systems including consensus, swarming, and formation (referring to Chu, Wang, Chen, and Mu (2006), Cortés (2006), Egerstedt and Hu (2001), Fax and Murray (2004), Gazi and Passino (2004), Hu and Hong (2007) and Martinez, Cortes, and Bullo (2007)).

In the studies of multi-agent coordination, two important problems are very interesting: target aggregation, which is concerned with how a group of agents move together to a target region, and state agreement, which talks about how a group of agents reach a consensus without a given target. Sometimes, target-oriented coordination can be formulated as a leader–follower problem with multiple (virtual) leaders, while state agreement can be described as a leaderless coordination problem. In fact, a "leader" in the multi-agent systems may be a special (informed) agent, or a moving target, or a reference node to guide the whole group. Although there is usually a single leader for the leader-following formulation in many existing results, multiple (virtual) leaders can be found or needed in multi-agent coordination. In fact, a multi-leader framework may be useful in many practical problems. For example, a simple model for the fish flocking was given to simulate foraging and demonstrate that, the larger the group, the smaller the proportion of "leaders" needed to guide the group to the food source in Couzin, Krause, Franks, and Levin (2005), while moving targets can be also viewed as multiple "leaders" in pursuit-evasion operations as in Oh, Schenato, Chen, and Sastry (2007). Lin, Francis, and Maggiore (2005) also discussed an interesting model for a group of agents with straight-line formation containing two "edge leaders", where all the agents converge to the line segment specified by the two edge leaders. On the other hand, state agreement, or sometimes called consensus or synchronization, has been studied in different research areas (for example, DeGroot (1974), Lynch (1997), Lin, Francis, and Maggiore (2007) and Olfati-Saber and Murray (2004)). Without targets or leaders given in advance, all the agents achieve a consensus and their states become the same by their intra-agent interactions or communications.

Variable interconnection topologies between mobile agents pose challenging problems in the studies of multi-agent systems because of the complexity resulting from time-varying and non-smooth structures. Many efforts have been made to handle

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multi-agent controls with dynamic topologies (Hong, Hu, & Gao, 2006; Olfati-Saber & Murray, 2004; Ren & Beard, 2005; Tanner, Jadbabaie, & Pappas, 2003). Among the research results, "joint connection" or related concepts play an important role in the investigation of multi-agent coordination. For example, Tsitsiklis, Bertsekas, and Athans (1986) studied the distributed asynchronous iterations, where a team of agents achieve consensus on a common value with possibly outdated values of their neighbors. Jadbabaie, Lin, and Morse (2003) proved the consensus of a simplified Vicsek model (proposed in Vicsek, Czirok, Jacob, Cohen, and Schochet (1995)) with joint-connection assumption in a similar way. Moreover, Hong, Gao, Cheng, and Hu (2007) investigated the jointly-connected coordination for second-order agent dynamics. However, this problem becomes much more difficult if the agent dynamics are nonlinear. Moreau discussed the stability and state agreement problems for nonlinear discrete-time agents with time-varying interconnection assumption on $[t, \infty)$ in Moreau (2005). For nonlinear continuous-time agent dynamics with jointly-connected interaction graphs, results seem even harder to be obtained. Lin et al. made a good start in this research direction and provided conditions to ensure the state agreement for directed multi-agent networks under uniform joint connectivity in Lin et al. (2007).

In this paper, we consider a group of continuous-time agents with variable intra-agent (communication) connection and nonlinear agent dynamics. Also, we investigate with some connectivity assumptions given for joint topology in $[t, \infty)$, the set stability of the networked agents by virtue of graph theory, convex analysis, and stability theory. By neighborhood rules, we show that a set of agents with nonlinear individual dynamics can flock to a convex target set, or achieve the state agreement of the whole group, in some important switching jointly-connected cases. To solve the problems, we propose a limit-set-based approach, different from those adopted in Moreau (2005) and Lin et al. (2007), to deal with the convergence in either target aggregation or state agreement of the considered multi-agent networks.

The paper is organized as follows. Section 2 introduces basic concepts and preliminary results, while Section 3 formulates our problems, target aggregation and state agreement. Then Section 4 studies the set stability of the considered multi-agent system with jointly-connected topologies and the structure of its limit set with a proposed analysis technique. Furthermore, Section 5 analyzes the convergence in some important target-aggregation cases. Then, Section 6 discusses the state agreement for multi-agent systems without target sets. Finally, Section 7 gives the concluding remarks.

2. Preliminaries

In this section, we introduce some preliminary knowledge for the following discussion.

First of all, we introduce some basic concepts and notations in graph theory (referring to Godsil and Royle (2001) for details). A directed graph (or digraph) is usually denoted as $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \dots, n\}$ is the set of nodes and $\mathcal{E}$ is the set of arcs, each element of which is an ordered pair of distinct nodes in $\mathcal{N}$. (i, j) denotes an arc leaving from node v_i (or simply i) and entering node v_j (or j). A walk in digraph $\mathcal{G}$ is an alternating sequence $i_1e_1i_2e_2 \cdots e_{k-1}i_k$ of nodes i_m and arcs $e_m = (i_m, i_{m+1}) \in \mathcal{E}$ for $m = 1, 2, \dots, k$. If there exists a walk from node i to node j then node j is said to be reachable from node i. In particular, each node is thought to be reachable by itself. A node v which is reachable from any node of $\mathcal{G}$ is called a globally reachable node of $\mathcal{G}$. $\mathcal{G}$ is said to be *quasi-strongly connected* if for every two nodes i and j there is a node k from which i and j are reachable. Given a digraph $\mathcal{G}$, its opposite graph $\mathcal{G}^*$ is the digraph formed by changing the orientation of each arc in $\mathcal{G}$. It is known that $\mathcal{G}$ is quasi-strongly connected if and only if $\mathcal{G}^*$ has a globally reachable node (Berge & Ghouila-Houri, 1965).

If $\mathcal{G}_1 = (\mathcal{V}, \mathcal{E}_1)$ and $\mathcal{G}_2 = (\mathcal{V}, \mathcal{E}_2)$ have the same node set, the union of the two digraphs is defined as $\mathcal{G}_1 \cup \mathcal{G}_2 = (\mathcal{V}, \mathcal{E}_1 \cup \mathcal{E}_2)$. A time-varying digraph is defined as $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$ with $\sigma : t \to \mathcal{Q}$ as a piecewise constant function, where $\mathcal{Q}$ is a finite set with all the possible digraphs with node set $\mathcal{V}$.

Additionally, $\mathcal{G}([t_1, t_2))$ denotes the joint digraph in time interval $[t_1, t_2)$ with $t_1 < t_2 \le +\infty$, that is,

$$\mathcal{G}([t_1, t_2)) = \bigcup_{t\in[t_1,t_2)} \mathcal{G}(t) = (\mathcal{V}, \bigcup_{t\in[t_1,t_2)} \mathcal{E}_{\sigma(t)}). \tag{1}$$

The graph $\mathcal{G}(t)$ is called "jointly quasi-strongly connected" in $[t_1, t_2)$ if its joint digraph $\mathcal{G}([t_1, t_2))$ is quasi-strongly connected. Moreover, if there is a constant $T_0 > 0$, such that $\mathcal{G}([t, t + T_0])$ is quasi-strongly connected for any t, then $\mathcal{G}_{\sigma(t)}$ is said to be *uniformly* quasi-strongly connected (with respect to T_0).

Next, we recall some notations in convex analysis (see Rockafellar (1972) for details). A set $K \subset R^m$ is said to be convex if $(1-\gamma)x + \gamma y \in K$ whenever $x \in K, y \in K$ and $0 < \gamma < 1$. For any set $S \subset R^m$, the intersection of all convex sets containing S is called the *convex hull* of S, denoted by $co(S)$. Particularly, the convex hull of a finite set of points $x_1, \dots, x_n \in R^m$ is a polytope, denoted by $co\{x_1, \dots, x_n\}$. We cite two lemmas on convex analysis, which can be found in Aubin and Cellina (1984).

Lemma 1 (*Best-Approximation Theorem*). *Let K be a closed convex subset of a Hilbert space X. We can associate to any $x \in X$ a unique element $\pi_K(x) \in K$ satisfying*

$$\|x - \pi_K(x)\| = \min_{y\in K} \|x - y\|,$$

where the map π_K is called the projector onto K. Moreover,

$$\langle \pi_K(x) - x, \pi_K(x) - y\rangle \le 0, \quad \forall y \in K.$$

Lemma 2. *Let K be a closed convex subset of a Hilbert space X and d_K the function defined on X by $d_K(x) \triangleq \inf\{\|x - y\| \mid y \in K\}$. Then $d_K^2(x) = \inf\{\|x - y\|^2 | y \in K\}$ is continuously differentiable and*

$$\nabla d_K^2(x) = 2(x - \pi_K(x)),$$

where $\nabla d_K^2(x)$ denotes the gradient of function d_K^2 at point x.

Then, we consider the Dini derivative for the following non-smooth analysis. Let a and b $(> a)$ be two real numbers and consider a function $h : (a, b) \to R$ and a point $t \in (a, b)$. The upper Dini derivative of h at t is defined as

$$D^+h(t) = \limsup_{s\to 0^+} \frac{h(t+s) - h(t)}{s}.$$

Obviously, when h is continuous on (a, b), h is non-increasing on (a, b) if and only if $D^+h(t) \le 0$ for any $t \in (a, b)$ (more details can be found in Rouche, Habets, and Laloy (1977)). The next result is given for the calculation of Dini derivative.

Lemma 3 (Danskin, 1966; Lin et al., 2007). *Let $V_i(t, x) : R \times R^n \to R$ be C^1 for $i = 1, 2, \dots, n$ and let $V(t, x) = \max_{i=1,2,\dots,n} V_i(t, x)$. If*

$$\mathcal{I}(t) = \{i \in \{1, 2, \dots, n\} : V(t, x(t)) = V_i(t, x(t))\}$$

is the set of indices where the maximum is reached at t, then

$$D^+V(t, x(t)) = \max_{i\in\mathcal{I}(t)} \dot{V}_i(t, x(t)).$$