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Tracking control for boundary controlled parabolic PDEs with varying parameters: Combining backstepping and differential flatness[☆]

Thomas Meurer*, Andreas Kugi

Automation and Control Institute, Complex Dynamical Systems Group, Vienna University of Technology, Vienna, Austria

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ABSTRACT

The combination of backstepping-based state-feedback control and flatness-based trajectory planning and feedforward control is considered for the design of an exponentially stabilizing tracking controller for a linear diffusion–convection–reaction system with spatially and temporally varying parameters and nonlinear boundary input. For this, in a first step the backstepping transformation is utilized to determine a state-feedback controller, which transforms the original distributed-parameter system into an appropriately chosen exponentially stable distributed-parameter target system of a significantly simpler structure. In a second step, the flatness property of the target system is exploited in order to determine the feedforward controller, which allows us to realize the tracking of suitably prescribed trajectories for the system output. This results in a systematic procedure for the design of an exponentially stabilizing tracking controller for the considered general linear diffusion–convection–reaction system with varying parameters, whose applicability and tracking performance is evaluated in simulation studies.

1. Introduction

Diffusion–convection–reaction processes occur in a large variety in chemical and biochemical engineering. Typical examples include fixed-bed tubular reactors for production or degradation, activated sludge processes, or catalytic converters for emission control and purification. Due to the strong interactions of the diffusive, the convective, and the reactive effects, a rather complex dynamical behavior with multiple or periodic stable and unstable steady states can evolve (see, e.g., Jensen and Ray (1982)), which requires to consider advanced model-based control strategies for the process operation.

Since the modeling of these systems usually leads to a description in terms of parabolic partial differential equations (PDEs), control design is in general either based on the early or the late lumping approach. In the early lumping approach, the system is approximated first and the controller design is performed based on the lumped model (Balas, 1986; Christofides, 2001; Georgakis, Aris, & Amundson, 1977). However, this often leads to high-dimensional and complex feedback control structures, which do not fully exploit the physical structure of the system. In addition, the neglected dynamics may even lead to the destabilization of the closed-loop system due to spillover effects (Balas, 1978). In the late lumping approach, the distributed nature of the system is kept as long as possible and the controller design is performed using the infinite-dimensional process model. Thereby, the functional analytic setting using semigroup theory and related concepts (see, e.g., Curtain and Zwart (1995) and Luo, Guo, and Morgül (1999) and the references therein for a modern and comprehensive overview) has proven to be a powerful tool for system analysis and feedback control design. Herein, the emphasis is mainly put on the extension of well-established concepts for finite-dimensional systems such as pole placement, robust, and optimal control (see, e.g., Curtain (1985), Curtain and Zwart (1995), Lasiecka and Triggiani (1983) and Schuhmacher (1983)). For the boundary control of parabolic PDEs, various general results exist for the design of stabilizing feedback control (see, e.g., Lasiecka and Triggiani (1983) and Nambu (1984)). However, in general it can be observed that the available results essentially rely on certain assumptions of the spectrum of the linear system operator. This either significantly complicates or prevents their applicability to distributed-parameter systems (DPSs) with time-varying parameters. In addition, the available extensions to nonlinear DPSs are so far rather limited (see, e.g., Luo et al. (1999)).

On the other hand, the recent extensions of the backstepping concept, well known for finite-dimensional nonlinear systems,

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* Corresponding author. Tel.: +43 1 58801 37627; fax: +43 1 58801 37699.
E-mail addresses: meurer@acin.tuwien.ac.at (T. Meurer), kugi@acin.tuwien.ac.at (A. Kugi).

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to boundary controlled linear parabolic PDEs have provided a promising and in particular systematic method for the design of stabilizing state-feedback controllers and observers (Krstic & Smyshlyaev, 2008; Liu, 2003; Smyshlyaev & Krstic, 2004). In addition to linear systems with constant or spatially varying parameters, first results on the incorporation of a single time-varying reaction parameter (Smyshlyaev & Krstic, 2005) as well as the extension to DPSs with nonlinearities in terms of Volterra series (Vazquez & Krstic, 2007) are available. Roughly speaking, the backstepping approach is based on the determination of the kernel of a Volterra integral equation to transform the possibly unstable DPS into a suitably selected exponentially stable target DPS. This, however, requires the solution of a higher-dimensional PDE governing the evolution of the kernel. Once the kernel is determined, the respective state-feedback controller realizing the desired transformation is immediately obtained.

In addition to the stabilization problem, which is far from being completely solved for DPSs, the consideration of the trajectory tracking control problem, i.e. the design of a control such that the outputs of the DPS follow prescribed desired reference trajectories, has recently gained more and more attraction. This is particularly due to the increasing demands on product quality and production efficiency, which require to turn away from the pure stabilization of an operating point towards the realization of specific start-up, transition or tracking tasks as can be observed in several industrial applications (see, e.g., Corriou (2004), Meurer, Thull, and Kugi (2008) and Petit, Rouchon, Boueilh, Guérin, and Pinvidic (2002)). When dealing with tracking control problems for finite-dimensional nonlinear systems, differential flatness has proven to be a powerful tool for system analysis, trajectory planning, and feedforward as well as feedback control (see, e.g., Fliess, Lévine, Martin, and Rouchon (1995), Rothfuß, Rudolph, and Zeitz (1996), Rudolph (2003) and Sira-Ramirez and Agrawal (2004) and the references therein). Roughly speaking, differential flatness denotes the ability to parametrize all system variables (states and inputs) by a so-called flat output and its time-derivatives up to a certain problem-dependent order. The parametrization of the system inputs by the flat output also corresponds to the determination of an inverse system representation in terms of the flat output. In the past years, this approach has been successfully extended to certain classes of infinite-dimensional systems (see, e.g., Meurer (2005) and Rudolph (2003)), where the system inversion is performed with respect to the governing PDEs to retain the inherent infinite-dimensional system structure. Examples include parabolic PDEs such as linear and nonlinear diffusion–convection–reaction systems (DCRSs) (Laroche, Martin, & Rouchon, 2000; Lynch & Rudolph, 2002; Meurer & Zeitz, 2005), Euler–Bernoulli beam models (Fliess, Mounier, Rouchon, & Rudolph, 1997; Meurer et al., 2008) or hyperbolic PDEs, like heavy chain systems (Petit & Rouchon, 2001; Thull, Wild, & Kugi, 2006), water tank systems (Petit & Rouchon, 2002), and Timoshenko beam models (Becker & Meurer, 2007; Woittennek & Rudolph, 2003).

In the following, backstepping and differential flatness are combined in an integrated tracking control design approach for unstable boundary controlled parabolic DPSs with spatially and temporally varying parameters. For this, the backstepping transformation is used to determine a state-feedback control, which maps the original DPS into an exponentially stable target DPS of a significantly simpler structure. Thereby, it is shown that a reformulation of the target DPS allows us to introduce an additional degree-of-freedom, which can be exploited for the flatness-based trajectory planning and feedforward control design to realize the tracking of suitably prescribed desired trajectories for the output of the original DPS with varying parameters. The contribution of this paper is threefold. On the one hand, it constitutes a first combination of backstepping and flatness-based control design methods to systematically determine exponentially stabilizing tracking controllers for parabolic DPSs with nonlinear boundary input. This is on the other hand complemented by the rigorous extension of the backstepping approach to systems with non-separable spatially and temporally varying reaction parameters. Finally, the backstepping transformation allows us to extend the available results on the flatness-based trajectory planning to parabolic DPSs with spatially and temporally varying parameters (see, e.g., Lynch and Rudolph (2002)) which are so far restricted to smooth coefficients allowing a power series representation of a certain Gevrey order.

The paper is organized as follows. In Section 2 the considered tracking control problem for a DCRS with spatially and temporally varying parameters and nonlinear boundary input is formulated. Based on a suitable change of coordinates, in Section 3 the backstepping transformation is applied to transform the governing DPS into an exponentially stable target system. The flatness property of the target system is exploited in Section 4 to determine the inverse system representation in terms of a flat or basic output. Combining the obtained backstepping-based state-feedback controller and the flatness-based feedforward controller results in an exponentially stabilizing tracking controller, which allows us to track appropriately prescribed output trajectories. This is demonstrated in Section 5, where simulation results confirm the applicability and the achievable high tracking performance of the proposed approach.

2. Boundary tracking control problem

Subsequently, a scalar linear DCRS with spatially and temporally varying parameters and nonlinear boundary input is considered. The PDE reads as

$$\partial_t x(z,t) = b(z)\partial_z^2 x(z,t) + c(z)\partial_z x(z,t) + d(z,t)x(z,t) \tag{1}$$

with domain $(z,t) \in (0,L) \times \mathbb{R}^+_{t_0}$, where $\mathbb{R}^+_{t_0} := \{t \in \mathbb{R}^+ \mid t > t_0\}$. The respective boundary conditions (BCs) are assumed as

$$-p_0\partial_z x(0,t) + p_1 x(0,t) = 0, \quad t > t_0 \tag{2}$$

$$\psi(x(L,t), \partial_z x(L,t)) = u(t), \quad t > t_0 \tag{3}$$

while the consistent initial condition (IC) follows as

$$x(z,t_0) = x_0(z), \quad z \in [0,L]. \tag{4}$$

Depending on the values of $p_0 \geq 0$, $p_1 \geq 0$, a Dirichlet ($p_0 = 0$, $p_1 = 1$), a Neumann ($p_0 = 1$, $p_1 = 0$), or a mixed BC ($p_0 \neq 0$, $p_1 \neq 0$) is obtained at $z = 0$. Furthermore note that the boundary input $u(t)$ at $z = L$ enters the system in a general nonlinear fashion governed by the continuous but not necessarily bounded functional $\psi(\cdot,\cdot)$, which combines the state and its gradient at the outlet. For the various results on the existence and uniqueness of solutions to (1)–(4) with the input $u(t)$ from a certain Banach space, the interested reader is referred to, e.g., Ahmed and Xiang (1996), Amann (1988), Ladyženskaja, Solonnikov, and Ural'ceva (1998) and Lions (1971). Note that in general the existence of a solution is restricted to a certain finite time interval due to the time-dependence of $d(z,t)$. In addition, depending on the growth of the functional $\psi(\cdot,\cdot)$, the nonlinear BC (3) might introduce a finite time blow-up of the solution.

Since (1) represents a parabolic PDE it follows that necessarily $0 < b_l \leq b(z) \leq b_u < \infty$ for all $z \in [0,L]$ with positive constants b_l and b_u. In order to specify further assumptions on the boundedness and the differentiability of the convection and reaction parameters $c(z)$ and $d(z,t)$, the notion of a Gevrey class is required (Rodino, 1993).