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Brief paper

Hamilton–Jacobi–Bellman formalism for optimal climate control of greenhouse crop[☆]

Ilya Ioslovich [*], Per-Olof Gutman, Raphael Linker

Faculty of Civil and Environmental Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel

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ABSTRACT

The paper describes a simplified dynamic model of a greenhouse tomato crop, and the optimal control problem related to the seasonal benefit of the grower. A HJB formalism is used and the explicit form of the Krotov–Bellman function is obtained for different growth stages. Simulation results are shown.

1. Introduction

The greenhouse horticulture sector is growing fast and is attaining greater economic and social importance. Many efforts have been made to develop advanced computerized greenhouse climate control. In particular, different interesting and important optimal control approaches have been proposed, see e.g. Aikman (1996), Ioslovich and Seginer (1995, 1998), Ioslovich and Tchamitchian (1998), Pucheta, Schugurensky, Fullana, Patino, and Kuchen (2006), Seginer (1989), Seginer and McClendon (1992), Seginer and Ioslovich (1998), Tchamitchian and Ioslovich (1998), Van Straten, Challa, and Buwalda (2000), Van Henten (1994) and Van Straten, van Willigenburg, and Tap (2002). However, optimal control concepts and nonlinear dynamic programming (NDP) in particular have almost not been used in practice, due to the complexity of implementation. This paper is an attempt to alleviate the complexity problem. Optimal control theory, see Pontryagin, Boltyanskii, Gamkrelidze, and Mishchenko (1962), makes it possible to "transform" a known weather sequence over the growing season to an optimal control sequence, by the simultaneous determination of state and costate variables, and replace the seasonal optimization by instantaneous optimization of the Hamiltonian function at each time moment. However, in addition to numerical difficulties, the details of the weather are hard to predict. Here we do not assume a detailed and correct weather forecast, but take an approach based on the so-called *climate index*, (Ioslovich & Gutman, 2005), and on a simplified crop growth model. Then the Krotov version of nonlinear dynamic programming (NDP), Krotov (1996), is used *off-line*, and even before the growing season, to determine the scalar *adjusted costate*, Seginer and Ioslovich (1998), which may be interpreted as the optimal control "intensity" [$/kg accumulated dry matter]. By knowing the value of the adjusted costate, the current weather measurement determines the instantaneous optimal control in an on-line optimization procedure, (Ioslovich & Seginer, 1998). Clearly, such a procedure is conceptually simple for the grower, and numerically tractable. The aim of this paper is to determine the optimal control intensity based on the three stage crop growth model and using the HJB formalism as the sufficient conditions of optimality. The paper is organized as follows: Section 2 contains the description of the model and the statement of the problem. Section 3 is devoted to a short description of the Krotov–Bellman sufficient conditions, (Krotov, 1996). Section 4 gives the description of the explicit Krotov–Bellman functions for the three growth stages. Simulation results are reported in Section 5, and the concluding remarks are found in Section 6.

2. Description of the simplified model and statement of the problem

The growth of the greenhouse tomato plant is described by two state variables which have different differential equations during different growth stages, (Ioslovich, Gutman, & Linker, 2007). These stages are: vegetative, vegetative–reproductive (mixed), and

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[*] Corresponding author. Tel.: +972 4 8294254; fax: +972 4 8295696.
E-mail address: agrilya@tx.technion.ac.il (I. Ioslovich).

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reproductive. The partition factor that determines the allocation of the accumulated dry matter to vegetative and reproductive parts of the plant is different in each stage. The model, MBM-A, was calibrated against the data from simulations of the multivariable detailed TOMGRO model, (Dayan et al., 1993). The state variables of the MBM-A are x and y with given initial conditions $x(0) = x_0,\ y(0) = y_0 = 0$. Here, x [kg d.m./m^2] is the accumulated vegetative dry mass, d.m., including green leaves, stems and green fruits. The variable y [kg d.m. /m^2] corresponds to the harvestable red fruits (economic end-product). $x_0 > 0$ stands for the seedlings obtained from the nursery. The steady state greenhouse model in Hwang and Jones (1994) gives the function $M(t, U)$ [kg/(day*m^2)], that represents the daily rate of dry weight accumulation due to net photosynthesis per unit of sunlit area, and depends on the current outdoor climate inputs at the moment t, and the vector of control variables U, including greenhouse heating, ventilation and CO_2 enrichment. The dimensionless coefficient k_{rv} represents the ratio between values of the photosynthesis radiation effectiveness of the reproductive and the vegetative organs. The dimensionless function $f(x)$ may be loosely defined as the "light interception factor", i.e. the fraction of light intercepted by the canopy. It can be approximately expressed as a function of the vegetative dry mass, namely,

$$f(x) = 1 - \exp(-\beta x). \tag{1}$$

The determination of the switching time between growth stages is done in terms of τ, *the effective degree-days, EDD*. The value of τ is a time integral of ET [°C], where ET is the effective temperature of the greenhouse crop canopy, i.e. the temperature above a given threshold T_l,

$$ET = \max\{0, (T - T_l)\}. \tag{2}$$

The time $t = t_f$ of the end of the season is fixed. The value $S(t, U)$ [kg d.m./(day m^2)] is equal to $ET\sigma^{-1}$, where $\sigma = 10^3$ [day (°C m^2)/(kg d.m.)] is a conversion factor due to the given units. Thus S is the effective temperature converted to the units of daily dry matter accumulation. For τ we have the equation

$$\frac{d\tau}{dt} = S(t, U)\sigma. \tag{3}$$

Similarly to Ioslovich and Gutman (2005) we assume that the following constant ratio holds,

$$M(t, U)/S(t, U) = K_c. \tag{4}$$

For open field crops the dimensionless coefficient K_c is a climate index that can be calculated from local climate data history. The mean daily temperature is strongly correlated with the mean daily light, and light is strongly correlated with photosynthesis. Therefore, a strong correlation between temperature and photosynthesis can be assumed, e.g. approximately as in (4). Thus a coefficient K_c, the *climate index*, is an integrated value to characterize the climate. For greenhouse crops this index corresponds to the source/sink activity balance constraint, see Tchamitchian and Ioslovich (1998). The growth is said to be 'balanced' if the photosynthetic source strength of carbon balances the sink strength of the effectively growing crop. In our greenhouse tomato case, the coefficient K_c can be extracted from TOMGRO, where this proportionality is clearly observed during balanced growth. The vegetative period is described by the equations

$$\frac{dx}{dt} = M(t, U)f(x), \qquad \frac{dy}{dt} = 0. \tag{5}$$

This period starts at $t = t_0$ and ends at $\tau = \tau_1$. However, using (3), (5) and (4) one can see that

$$\frac{dx}{d\tau} = f(x)K_c\sigma. \tag{6}$$

Thus it is easy to calculate the value $x(\tau = \tau_1) = x_1$ which is independent of $U(t)$. Therefore the end of the vegetative period is determined by the moment when $x(t) = x_1$. In the intermediate (mixed) vegetative–reproductive stage the rate of growth of the red fruits is limited by the potential sink demand of the reproductive organs, and the equations of the process are

$$\frac{dx}{dt} = M(t, U)(1 - \alpha)f(x), \qquad \frac{dy}{dt} = M(t, U)g(y)/K_c. \tag{7}$$

First Eq. (7) is calibrated such that the vegetative dry matter at the end of the mixed period $x(\tau = \tau_2)$ is equal to the value found in TOMGRO. Second Eq. (7) is equivalent to

$$dy/dt = S(t, U)g(y). \tag{8}$$

Similarly to $f(x)$, $g(y)$ is assumed to be a smooth increasing dimensionless function,

$$g(y) = \epsilon + \nu[1 - \exp(-\gamma y)].$$

The coefficients $\epsilon,\ \nu,\ \alpha$, all dimensionless, and γ [m^2/kg d.m.], are extracted from TOMGRO simulations. The end of the mixed period at $\tau = \tau_2$, found from TOMGRO, can be restated as the condition $x = x_2$, where x_2 does not depend on the control sequence leading to it. From (7), (3) and (8) we have

$$\frac{dx}{d\tau} = (1 - \alpha)f(x)K_c, \qquad \frac{dy}{d\tau} = g(y)\sigma. \tag{9}$$

We recall that the values $x(\tau_1) = x_1$ and $y(\tau_1) = y_1 = 0$ are already known, and notice from (9) that the values $x_2 = x(\tau_2)$ and $y(\tau_2) = y_2$ can be easily determined. It will be shown below that $x_2(\tau_2) = x(t_f)$, thus the value x_2 is a boundary condition for the variable x at the final time t_f. During the third period (the reproductive stage) all the assimilates are directed to the reproductive organs, and the state equations become

$$\frac{dx}{dt} = 0, \qquad \frac{dy}{dt} = M(t, U)f(x)\eta. \tag{10}$$

Here the notation used is

$$\eta = k_{rv}\theta\xi. \tag{11}$$

The dimensionless constant value θ represents the loss coefficient of the dry weight allocated to fruits, due to different factors such as fruit abortions, *etc.* The overall fruit loss coefficient η for the reproductive stage is a product of k_{rv}, θ, and ξ, where the additional coefficient $\xi < 1$ is added to reflect the fact that some photosynthetic assimilates are used to compensate for dying leaves, etc. With this approximation we see that x remains constant, while y is growing linearly. The reproductive period ends at the given final time $t = t_f$. The final value $x(t_f) = x_2$ is a fixed boundary condition at the end of the trajectory. The performance criterion (the objective of the optimal control problem) is

$$Q = c_r y(t_f) - \int_{t_0}^{t_f} q(t, U)dt \to \max \tag{12}$$

which represents the maximization of the grower's monetary net income, i.e. the difference between the sales price of the harvestable (red) fruits and the cost of the greenhouse operation, $\int_{t_0}^{t_f} q(t, U)dt$. Here c_r [\$/(kg d.m.)] is the unit price of red fruits. The cost $q(t, U)$ is determined as

$$q(t, U) = c_h h + c_C C, \tag{13}$$

where h [J/day/m^2] is the heating, and C [kg CO_2/day/m^2] is the CO_2 enrichment control fluxes, respectively, and c_h [\$/J/m^2], c_C [\$/kg/m^2] are the corresponding unit prices. The objective function (12) contains a function of the final state, and an integral part; thus it is a so-called Bolza problem, see Goldstine (1980). Due to the three different growth periods, one may define different regions in the state space (phase plane) which characterize the solution of the differential equations (5), (7) and (10), illustrated in Fig. 1. The regions are: