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Brief paper

A GES attitude observer with single vector observations[☆]

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ABSTRACT

This paper proposes a globally exponentially stable (GES) observer for attitude estimation based on a single time-varying reference vector, in inertial coordinates, and corresponding vector, in body-fixed coordinates, in addition to angular velocity readings. The proposed solution is computationally efficient and, in spite of the fact that the observer does not evolve on the Special Orthogonal Group $SO(3)$, an explicit solution on $SO(3)$ is also provided, whose error is shown to converge exponentially fast to zero for all initial conditions. The distinct roles of the inertial and the corresponding body-fixed vectors on the observability of the system are also examined and simulation results are shown that illustrate the performance of the proposed attitude observer in the presence of low-grade sensor specifications.

1. Introduction

Attitude estimation has been a hot topic of research in the past decades, see e.g. Batista, Silvestre, and Oliveira (2009), Mahony, Hamel, and Pflimlin (2008), Metni, Pflimlin, Hamel, and Soueres (2006), Rehbinder and Ghosh (2003), Sanyal, Lee, Leok, and McClamroch (2008), Tayebi, McGilvray, Roberts, and Moallem (2007), Thienel and Sanner (2003), Vasconcelos, Cunha, Silvestre, and Oliveira (2010) and the references therein. The reader is referred to Crassidis, Markley, and Cheng (2007) for a survey on the topic. However, only recently has attitude estimation been studied based on time-varying reference vectors and, in particular, single vector observations, see Kinsey and Whitcomb (2005, 2007), Lee, Leok, McClamroch, and Sanyal (2007), and Mahony, Hamel, Trumpf, and Lageman (2009). In Akella, Seo, and Zanetti (2006), an explicit solution on $SO(3)$ is proposed with almost globally asymptotically stable error dynamics.

This paper presents a novel theoretical attitude estimation framework based on single vector observations. Applications include, e.g., attitude estimation of unmanned vehicles that depend on electromagnetic or acoustic feedback of direction vectors of known landmarks. For space applications, it is interesting e.g. in attitude estimation in orbits with gravity gradient effects or even using magnetometers and sun sensor readings, as the corresponding inertial vectors are slowly time-varying. Alternative applications include sensor calibration, see e.g. Kinsey and Whitcomb (2005) and Kinsey and Whitcomb (2007). Dual to the topic of attitude estimation is attitude stabilization, see e.g. Wen and Kreutz-Delgado (1991), Campolo et al. (2009), and Sanyal, Fosbury, Chaturvedi, and Bernstein (2009) and the references therein. An interesting separation principle can be found in Maithripala, Berg, and Dayawansa (2006) and Maithripala, Berg, and Dayawansa (2005).

The main contribution of this paper is the development of a novel attitude observer based on single vector observations with globally exponentially stable error dynamics. Central to the observer design is the construction of a set of auxiliary reference vectors (and corresponding vectors in body-fixed coordinates) and the derivation of sufficient observability conditions, which result in appropriate persistent excitation conditions that also allow for norm changes, including null vectors for some time. The proposed design is computationally efficient and the stability analysis builds on well-established Lyapunov results and linear systems theory. Unlike prior contributions that resort to local coordinates, the unit quaternion, or Lie group techniques, the approach followed in this paper consists in the embedding of $SO(3)$ into $\mathbb{R}^9$, considering the problem in an unconstrained fashion. As such, widely known issues such as singularities, unwinding phenomena, slow convergence

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near unstable equilibrium points or topological limitations for achieving global stabilization using smooth feedback on $SO(3)$ do not apply, see Bhat and Bernstein (2000), Chaturvedi, Sanyal, and McClamroch (2011), Sanyal et al. (2009) and Wen and Kreutz-Delgado (1991) and the references therein. As the observers do not explicitly evolve on $SO(3)$, an explicit solution on $SO(3)$ is also provided resorting to a projection operator, an approach that has been employed previously in the design of interpolation methods on $SE(3)$, see Belta and Kumar (2002). The error of this solution is shown to converge exponentially fast to zero for all initial conditions.

The paper is organized as follows. The attitude kinematics and some preliminary definitions are given in Section 2, while the problem addressed in the paper is described in Section 3. The design and stability analysis of the attitude observer is presented in Section 4. In addition, the roles of the inertial and body-fixed vectors are discussed, as well as some refinements to the proposed solution. Simulation results that illustrate the achievable performance are presented in Section 5 and, finally, Section 6 summarizes the main contributions and conclusions of the paper.

1.1. Notation

Throughout the paper, the symbol $\mathbf{0}$ denotes a matrix (or vector) of zeros and $\mathbf{I}$ an identity matrix, both of appropriate dimensions. A block diagonal matrix is represented as $\text{diag}(\mathbf{A}_1, \dots, \mathbf{A}_n)$. For $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$, $\mathbf{x} \times \mathbf{y}$ represents the cross product.

2. Preliminaries

Let $\{I\}$ be an inertial reference frame, $\{B\}$ a body-fixed reference frame, and $\mathbf{R}(t) \in SO(3)$ the rotation matrix from $\{B\}$ to $\{I\}$. The attitude kinematics are given by $\dot{\mathbf{R}}(t) = \mathbf{R}(t)\mathbf{S}[\boldsymbol{\omega}(t)]$, where $\boldsymbol{\omega}(t) \in \mathbb{R}^3$ is the angular velocity of $\{B\}$, expressed in $\{B\}$, and $\mathbf{S}(.)$ is the skew-symmetric matrix such that $\mathbf{S}(\boldsymbol{x})\boldsymbol{y} = \boldsymbol{x} \times \boldsymbol{y}$. The angular velocity is assumed to be a continuous bounded signal, available for observer design purposes.

The following definitions are useful in the sequel.

Definition 1. A continuous norm-bounded vector $\mathbf{a}(t) \in \mathbb{R}^3$ is called persistently non-constant if there exist $\alpha > 0$, $\epsilon > 0$, and $\delta > 0$ such that, for all $t \geq t_0$ and $\mathbf{d} \in \mathbb{R}^3$, $\|\mathbf{d}\| = 1$, it is true that $\|\mathbf{a}(t)\| > \epsilon$ and

$$\int_t^{t+\delta} \|\mathbf{a}(\sigma) \times \mathbf{d}\| d\sigma \geq \alpha.$$

Definition 2. A set of N piecewise continuous norm-bounded vectors $\mathcal{A} = \{\mathbf{a}_i(t) \in \mathbb{R}^3, i = 1, \dots, N\}$ is called persistently non-collinear if there exist $\alpha > 0$ and $\delta > 0$ such that, for all $t \geq t_0$, there exist l and m such that

$$\int_t^{t+\delta} \|\mathbf{a}_l(\sigma) \times \mathbf{a}_m(\sigma)\| d\sigma \geq \alpha.$$

Definition 3. A set of N piecewise continuous norm-bounded vectors $\mathcal{A} = \{\mathbf{a}_i(t) \in \mathbb{R}^3, i = 1, \dots, N\}$ is called persistently non-planar if there exist $\alpha > 0$ and $\delta > 0$ such that, for all $t \geq t_0$ and $\mathbf{d} \in \mathbb{R}^3$, $\|\mathbf{d}\| = 1$, there exist l, m, and n such that

$$\int_t^{t+\delta} \|\mathbf{M}(\sigma)\mathbf{d}\| d\sigma \geq \alpha,$$

where $\mathbf{M}(t) := [\mathbf{a}_l(t) \quad \mathbf{a}_m(t) \quad \mathbf{a}_n(t)]^T \in \mathbb{R}^{3\times 3}$.

3. Problem statement

Consider a persistently non-constant reference vector $\mathbf{r}_1(t) \in \mathbb{R}^3$, expressed in inertial coordinates, and the corresponding vector $\mathbf{v}_1(t) \in \mathbb{R}^3$, expressed in body-fixed coordinates, that satisfies

$$\mathbf{r}_1(t) = \mathbf{R}(t)\mathbf{v}_1(t). \tag{1}$$

Suppose that the angular velocity $\boldsymbol{\omega}(t)$ is continuous and bounded. The problem of attitude estimation considered in the paper is that of designing an observer for the rotation matrix $\mathbf{R}(t)$ with globally exponentially stable error dynamics based on $\mathbf{v}_1(t)$, $\mathbf{r}_1(t)$, and $\boldsymbol{\omega}(t)$.

4. Observer design and stability analysis

The attitude observer proposed in the paper follows by embedding $SO(3)$ in $\mathbb{R}^9$, discarding the topological structure of the Special Orthogonal Group $SO(3)$. In order to simplify the derivation, consider a column stacking of $\mathbf{R}(t)$ given by $\boldsymbol{x}(t) = [\mathbf{z}_1^T(t) \quad \mathbf{z}_2^T(t) \quad \mathbf{z}_3^T(t)]^T \in \mathbb{R}^9$, where

$$\mathbf{R}(t) = \begin{bmatrix} \mathbf{z}_1^T(t) \\ \mathbf{z}_2^T(t) \\ \mathbf{z}_3^T(t) \end{bmatrix}, \quad \mathbf{z}_i(t) \in \mathbb{R}^3,\ i = 1, \dots, 3.$$

It is straightforward to show that $\dot{\boldsymbol{x}}(t) = -\mathbf{S}_3[\boldsymbol{\omega}(t)]\boldsymbol{x}(t)$, where $\mathbf{S}_3(\mathbf{x}) := \text{diag}(\mathbf{S}(\mathbf{x}), \mathbf{S}(\mathbf{x}), \mathbf{S}(\mathbf{x})) \in \mathbb{R}^{9\times 9}$, $\mathbf{x} \in \mathbb{R}^3$. An attitude observer for a persistently non-planar set of reference vectors is derived in Section 4.1. The observer for single vectors follows in Section 4.2 by constructing a persistently non-planar set of reference vectors in inertial coordinates (and corresponding body-fixed vectors) from a single persistently non-constant reference vector. The solutions provided by these observers converge asymptotically to $SO(3)$ but do not necessarily evolve on $SO(3)$. In Section 4.3, an explicit solution on $SO(3)$ is provided and examined, while the distinct roles of the reference vectors and body-fixed vectors are analysed in Section 4.4. Finally, in Section 4.5, additional discussion on the proposed observers is offered.

4.1. Persistently non-planar reference vectors

Consider a set of vectors $\mathcal{V} = \{\mathbf{v}_i(t) \in \mathbb{R}^3, i = 1, \dots, N\}$ expressed in body-fixed coordinates associated with a set of vectors $\mathcal{R} = \{\mathbf{r}_i(t) \in \mathbb{R}^3, i = 1, \dots, N\}$ expressed in inertial coordinates, such that $\mathbf{r}_i(t) = \mathbf{R}(t)\mathbf{v}_i(t)$, $i = 1, \dots, N$. Then, it is straightforward to show that $\mathbf{v}(t) = \mathbf{C}(t)\boldsymbol{x}(t)$, where $\mathbf{v}(t) := [\mathbf{v}_1^T(t) \quad \dots \quad \mathbf{v}_N^T(t)]^T \in \mathbb{R}^{3N}$ and

$$\mathbf{C}(t) := \begin{bmatrix} r_{11}(t)\mathbf{I}_3 & r_{12}(t)\mathbf{I}_3 & r_{13}(t)\mathbf{I}_3 \\ & \vdots & \\ r_{N1}(t)\mathbf{I}_3 & r_{N2}(t)\mathbf{I}_3 & r_{N3}(t)\mathbf{I}_3 \end{bmatrix} \in \mathbb{R}^{3N\times 9},$$

with $\mathbf{r}_i(t) = [r_{i1}(t)\ r_{i2}(t)\ r_{i3}(t)]^T \in \mathbb{R}^3,\ i = 1, \dots, N$.

Consider the attitude observer given by

$$\dot{\hat{\boldsymbol{x}}}(t) = -\mathbf{S}_3[\boldsymbol{\omega}(t)]\hat{\boldsymbol{x}}(t) + \mathbf{C}^T(t)\mathbf{Q}[\mathbf{v}(t) - \mathbf{C}(t)\hat{\boldsymbol{x}}(t)], \tag{2}$$

where $\mathbf{Q} = \mathbf{Q}^T \in \mathbb{R}^{3N\times 3N}$ is a positive definite matrix, and define the error variable $\tilde{\boldsymbol{x}}(t) := \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$. Then, the observer error dynamics are given by

$$\dot{\tilde{\boldsymbol{x}}}(t) = \mathcal{A}(t)\tilde{\boldsymbol{x}}(t), \tag{3}$$

where $\mathcal{A}(t) := -(\mathbf{S}_3[\boldsymbol{\omega}(t)] + \mathbf{C}^T(t)\mathbf{Q}\mathbf{C}(t))$.

The following theorem is the main result of this section.