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$\mathscr{H}_2$ robust filter design with performance certificate via convex programming☆

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Abstract

In this paper a new approach to $\mathscr{H}_2$ robust filter design is proposed. Both continuous- and discrete-time invariant systems subject to polytopic parameter uncertainty are considered. After a brief discussion on some of the most expressive methods available for $\mathscr{H}_2$ robust filter design, a new one based on a performance certificate calculation is presented. The performance certificate is given in terms of the gap produced by the robust filter between lower and upper bounds of a minimax programming problem where the $\mathscr{H}_2$ norm of the estimation error is maximized with respect to the feasible uncertainties and minimized with respect to all linear, rational and causal filters. The calculations are performed through convex programming methods developed to deal with linear matrix inequality (LMI). Many examples borrowed from the literature to date are solved and it is shown that the proposed method outperforms all other designs.

Keywords: Robust $\mathscr{H}_2$ filtering; Convex programming; Kalman filter; Filter design; Linear matrix inequalities (LMIs)

1. Introduction

Over the past years, many researchers have devoted a great deal of attention to the design of robust filters for LTI systems subject to parameter uncertainty. Two classes of parameter uncertainty appeared namely, norm bounded and convex bounded uncertainties, the last one originating the well known class of polytopic systems to be treated in this paper. The main difficulty for robust filter design stems from the necessity to determine an unique linear filter able to cope with different models generated by a set of uncertain parameters, keeping the estimation error norm below some guaranteed level, (Jain, 1975). Many contributions are available dealing with continuous-time (Geromel, 1999; Li, Luo, Davidson, Wong, & Bossé, 2002; Scherer & Köse, 2006; de Souza & Trofino, 1999; Tuan, Apkarian, & Nguyen, 2001; Xie & Soh, 1994) and discrete-time (Geromel, de Oliveira, & Bernussou, 2002; Shaked, Xie, & Soh, 2001; Theodor & Shaked, 1996; Xie, Soh, & Du, 1999) systems, among others. All of them, but (Scherer & Köse, 2006), share the following basic characteristics:

(a) the order of the robust filter is equal to the order of the plant,
(b) performance is not certificated.

Indeed, (a) is imposed as an instrumental condition to keep the design problem convex. In addition, the robust filter is determined from the minimization of a guaranteed cost, actually an upper bound of the true performance index, which does not provide any information about the distance to the true optimal filter. In other words, no information about the degree of sub-optimality is given. Hence, optimality is not theoretically certificated. Concerning (b), each new proposal is compared to the previous ones by means of simulation and performance determination for specific selected examples. Very recently a new important result on this area appeared (Scherer & Köse, 2006). The authors have developed a new $\mathscr{H}_2$ robust filter

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design method which eliminates both drawbacks we have just discussed. The robust filter is not restricted to have the same order of the plant and global optimality is assured. The method follows from an adequate description of the uncertainty by means of a class of multipliers. This very interesting theoretical result has, however, a difficulty which consists on how to fix the multiplier dynamics and its order for a particular design problem to be solved. As (Scherer & Köse, 2006) shows, the optimal filter performance depends on these multiplier characteristics.

For systems with known parameters, the minimization of the $\mathscr{H}_2$ norm of the estimation error provides the celebrated Kalman filter which is linear and, as a direct consequence of the optimality conditions, has the same order of the plant (Anderson & Moore, 1979; Colaneri, Geromel, & Locatelli, 1997). To deal with parameters uncertainty, the optimal filter is characterized by the equilibrium solution of a minimax optimization problem which can be interpreted as a Man–Nature game (for more detail on this aspect the reader is requested to see Martin & Mintz, 1983). The Nature selects the uncertain parameter by maximizing the $\mathscr{H}_2$ norm of the estimation error produced by a filter fixed by Man which has been selected by minimizing the same norm of the estimation error produced by a parameter fixed by Nature. The equilibrium solution (if any) provides the best robust filter and the worst uncertainty. Only in some especial cases the best filter is a Kalman filter associated to the worst uncertainty, (Geromel & Regis, 2006; Poor, 1980). Unfortunately, in the general case, such an equilibrium solution is extremely difficulty to calculate and only recently its existence has been proven for a particular class of polytopic parameter uncertainty (Geromel & Regis, 2006). Due to this, in the general case, it is not yet known the order of the optimal filter and it is not even known if it is finite but, the results of Geromel and Regis (2006) suggest that the order of the optimal filter is greater than the order of the plant.

In this paper, continuous- and discrete-time systems with parameter uncertainty of polytopic type are considered. The equilibrium solution of the already mentioned Man–Nature game is not exactly calculated but lower and upper bounds of the equilibrium $\mathscr{H}_2$ cost are provided as a way to certify the optimality gap and, by consequence, the distance from a particular filter to the optimal robust filter. The lower bound is optimized and yields a filter of order, prior to eventual zeros and poles cancellations, much greater than the order of the plant. Based on the result of this first step, a robust filter is determined. An upper bound and, consequently, the optimality gap are determined to certify the performance of the proposed robust filter with respect to the optimal one.

As a result, the order of the robust filter is, putting aside eventual poles and zeros cancellations, equal to the order of the plant times the number of vertices of the convex polytopic domain. With this respect two important points should be noticed. To our best knowledge, the first method available in the literature able to design a higher order filter (comparing to the order of the plant) from the solution of a convex programming problem expressed in terms of pure LMIs was proposed in Geromel and Regis (2006). The present paper generalizes the results of Geromel and Regis (2006) to cope with general polytopic systems. Second, the greater order of the proposed filter with respect to that of the plant appears to be essential to reduce conservatism yielding more accurate results when compared to the previous robust filter design procedures. As the examples solved indicate in many cases we obtain the optimal or, at least, a near-optimal robust filter.

The paper is organized as follows. In the next section the problem to be dealt with is stated and previous results on $\mathscr{H}_2$ robust filtering are discussed. In Section 3 a lower bound on the equilibrium solution of the Man–Nature game is proposed and its determination by means of LMIs is analyzed. Section 4 is devoted to determine a robust filter and an upper bound of the equilibrium cost. In Section 5 a great number of examples borrowed from the literature are solved and performances are compared. Both continuous- and discrete-time systems are considered. In Section 6 a more realistic practical application consisting on the estimation of the displacement of a tapered bar is presented. Models of increasing orders are considered to evaluate the proposed robust filter performance. Finally, Section 7 contains the conclusion and final remarks.

The notation used throughout is standard. Capital letters denote matrices and small letters denote vectors. For scalars, small Greek letters are used and $\mathbb{N} = \{1, \ldots, N\}$. For real matrices or vectors $(')$ indicates transpose. For square matrices $\mathrm{Tr}(X)$ denotes the trace function of X being equal to the sum of its eigenvalues and, for the sake of easing the notation of partitioned symmetric matrices, the symbol $(\bullet)$ denotes generically each of its symmetric blocks. For matrices or transfer functions U_λ denotes the linear parameter dependence $U_\lambda = \sum_i \lambda_i U_i$. Finally, the same notation

$$G(\zeta) = C(\zeta I - A)^{-1}B + D = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{1}$$

is used either for transfer functions of continuous- or discrete-time systems, where the real matrices A, B, C and D of compatible dimensions define a possible state space realization. With no ambiguity, for continuous-time systems, $G(\omega)$ denotes $G(\zeta)$ calculated at $\zeta = \mathrm{j}\omega$ and for discrete-time systems $G(\omega)$ denotes $G(\zeta)$ calculated at $\zeta = \mathrm{e}^{\mathrm{j}\omega}$ where, in both situations, $\omega \in \mathbb{R}$. For any real signal ξ, defined in the continuous- or discrete-time domain, $\hat{\xi}$ denotes its Laplace or $\mathscr{Z}$ transform, respectively.

2. Preliminaries and problem statement

Fig. 1 shows the basic structure of the filtering design problem in terms of the indicated transfer functions. From the exogenous signal $\hat{w}$, the transfer function $H(\omega)$ generates the

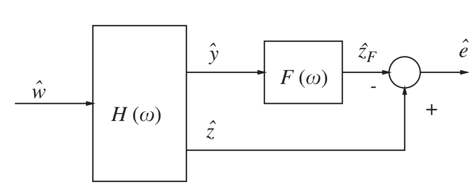

Fig. 1. Signal filtering structure.