Automatica 46 (2010) 990–998

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Input design for structured nonlinear system identification[☆]

Tyrone L. Vincent[a,*], Carlo Novara[b], Kenneth Hsu[c], Kameshwar Poolla[d]

[a] Division of Engineering, Colorado School of Mines, Golden, CO 80401, United States
[b] Dipartimento di Automatica e Informatica, Politecnico di Torino, Italy
[c] Grantham, Mayo, van Otterloo and Co. LLC 2150 Shattuck Avenue, Suite 900, Berkeley, CA 94704, United States
[d] Department of Mechanical Engineering, University of California, Berkeley, CA 94720, United States

ARTICLE INFO

Article history:
Received 2 December 2008
Received in revised form
24 November 2009
Accepted 23 February 2010
Available online 31 March 2010

Keywords:
Experiment design
Linear/nonlinear models
Identification methods

ABSTRACT

This paper is concerned with the input design problem for a class of structured nonlinear models. This class contains models described by an interconnection of known linear dynamic systems and unknown static nonlinearities. Many widely used model structures are included in this class. The model class considered naturally accommodates *a priori* knowledge in terms of signal interconnections. Under certain structural conditions, the identification problem for this model class reduces to standard least squares. We treat the input design problem in this situation.

An expression for the expected estimate variance is derived. A method for synthesizing an informative input sequence that minimizes an upper bound on this variance is developed. This reduces to a convex optimization problem. Features of the solution include parameterization of the expected estimate variance by the input distribution, and a graph-based method for input generation.

1. Introduction

Advances in networks, embedded processors, sensors, and actuators have led to complex systems that are composed of a large number of interconnected subsystems. Since these interconnected subsystems are often unknown or hard to model, identification of the elements in the network is a necessary first step toward control, signal estimation, or fault detection. A theory of system identification of interconnected systems has been developed for the special case where the subsystems consist of known linear dynamic blocks and unknown static nonlinearities. An example interconnected system is shown in Fig. 1. The elements $\mathcal{L}_i$ are known time invariant linear dynamic systems while $\mathcal{N}_i$ are unknown static nonlinearities to be identified. The problem of identifying the unknown static nonlinearities is called the structured identification problem (Claassen, 2001; Hsu et al., 2005a; Hsu, Vincent, Novara, Milanese, & Poolla, 2005b; Hsu, Poolla, & Vincent, 2008; Wemhoff, Packard, & Poolla, 1999). This is a related but distinct formulation from the well known Hammerstein, Wiener and LNL block oriented identification problems (Billings & Fakhouri, 1978; Narendra & Gallman, 1966), as in our case the linear blocks are known and the allowable interconnections are more general. In this paper, the input design problem for this class of interconnected nonlinear systems is considered.

The data structure used in the structured identification problem is a Linear Fractional Transformation (LFT) (Packard & Doyle, 1993). As shown in Fig. 2, all the linear dynamics are collected into the block $\mathcal{L}$. The static nonlinear functions are gathered into the block $\mathcal{N}$. All signals may be vector valued.

The linear block $\mathcal{L}$ is partitioned conformably as

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_{yu} & \mathcal{L}_{ye} & \mathcal{L}_{yw} \\ \mathcal{L}_{zu} & \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix}.$$

The nonlinear block $\mathcal{N}$ has a block-diagonal structure that we represent as

$$\mathcal{N} = \begin{bmatrix} \mathcal{N}_1 & & \\ & \ddots & \\ & & \mathcal{N}_n \end{bmatrix}. \quad (1)$$

This notation signifies a specific association of some components of z as inputs to nonlinearity $\mathcal{N}_i$. Without loss of generality, we assume that each component $\mathcal{N}_k$ is single-output. The formulation allows for repeated elements of $\mathcal{N}$. That is, it may be known *a priori* that two or more elements of $\mathcal{N}$ are identical.

[☆] Supported in part by the UC Discovery Grant ele07-10283 under the IMPACT program and by the NSF under Grants ECS 03-02554 and ECS 01-34132. The material in this paper was presented at the 2009 IFAC Symposium on System Identification, July 6–8, Saint-Malo, France. This paper was recommended for publication in revised form by Associate Editor Martin Enqvist under the direction of Editor Torsten Söderström.

[*] Corresponding author. Tel.: +1 303 273 3641; fax: +1 303 273 3602.
E-mail addresses: tvincent@mines.edu (T.L. Vincent), carlo.novara@polito.it (C. Novara), kennethhsu@gmail.com (K. Hsu), poolla@jagger.me.berkeley.edu (K. Poolla).

0005-1098/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2010.02.029

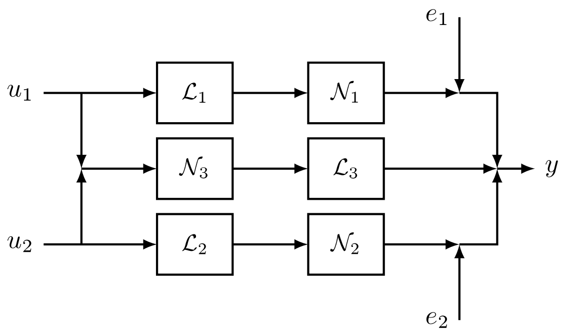

Fig. 1. Example of an interconnected system.

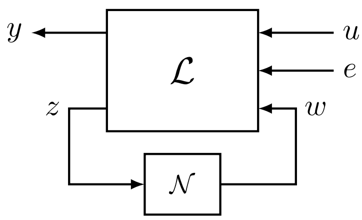

Fig. 2. LFT model structure.

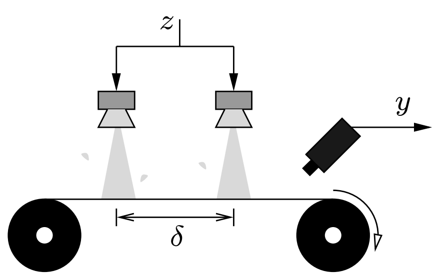

Fig. 3. Spray deposition system.

1.1. Motivating examples

In order to clarify the contribution of this paper, we provide some motivating examples.

Multi-zone deposition system: The manufacture of photovoltaics involves the deposition of thin films using co-evaporation sources (Hilt, Vincent, Joshi, & Simpson, 2006; Junker, Birkmire, & Doyle, 2004). In this continuous deposition process, a substrate unrolls into a vacuum chamber. Metals are heated to high temperature, creating a metal plume, which deposits on the substrate. Typically, several different types of metals are deposited over multiple zones of the chamber, but the same metal may be deposited in separate zones. Measurement occurs only at the exit of the chamber, and only determines the total amount of each metal deposited, not the relative amount from a particular zone. A simple schematic of this process with two zones and a single metal is shown in Fig. 3. The sources are separated by a known distance δ, and the total deposited material y is measured. The sources are typically controlled to a temperature set-point. As the substrate moves very slowly, the dynamics of this set-point control can be neglected. However, the mapping from temperature to deposition rate is highly nonlinear. Since the speed of the substrate is known, this system can be modeled as a Hammerstein system with a one-input/two-output static nonlinearity followed by a known linear system. Note however that this Hammerstein structure is non-standard, since the overall system is single input, single output, but there are two internal signals connecting the nonlinear and linear block.

The LFT model structure for this system contains the blocks

$$\mathcal{L} = \left[\begin{array}{c|c|cc} 0 & 1 & 1 & \xi^{-k} \\ \hline 1 & 0 & 0 & 0 \end{array}\right] \quad \text{and} \quad \mathcal{N} = \begin{bmatrix} f \\ g \end{bmatrix}$$

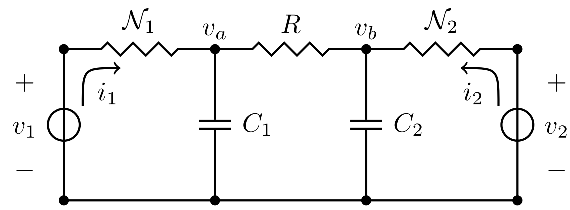

Fig. 4. Nonlinear circuit.

where f and g are the static actuator mappings from temperature to deposition rate, ξ^{-1} is the unit delay operator, and k is the number of samples taken when the substrate travels a distance of δ.

NARX model structure: By using a linear part that generates delayed inputs and outputs, our problem formulation encompasses general NARX model structures. Of course, it is also possible to specify interconnection structure within this class. An example of a structured NARX model is described by the difference equation

$$y_{k+1} = \mathcal{N}_1(y_k) + \mathcal{N}_2(y_{k-1}) + \mathcal{N}_3(u_k) + \mathcal{N}_4(u_{k-1}) + e_k.$$

Since the inputs to the nonlinearities are delayed versions of the inputs and outputs, this can also be put into LFT form, with the dynamic system $\mathcal{L}$ defining the required delays.

Circuit with nonlinear resistors: Consider the electric circuit shown in Fig. 4. This circuit contains two nonlinear elements with their behavior denoted by $v_{ni} = \mathcal{N}_i(i_{ni})$, where i_{ni} is the current through and v_{ni} is the voltage across the ith element. The resistor R and capacitors C_1 and C_2 are known, or identified using a small signal experiment. If we apply i_1 and i_2 as inputs, and measure v_1 and v_2, the LFT model structure is

$$\mathcal{L} = \left[\begin{array}{cc|cc|cc} Z_{11}(\xi) & -Z_{12}(\xi) & 1 & 0 & 1 & 0 \\ -Z_{12}(\xi) & Z_{22}(\xi) & 0 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right],$$

where the $Z_{ij}(\xi)$ define the impedance of the inner linear circuit and $\mathcal{N}$ is two by two and "diagonal". Besides the obvious application to electrical circuits, a thermal circuit with the same structure has been used to model building walls, floors and ceilings in building control applications (Lee & Braun, 2008).

1.2. Key assumptions

The LFT model structure captures arbitrary interconnected nonlinear systems. To make our problems more tractable we have to limit this class. We do so by making some key assumptions on the signal z.

Our first assumption ensures that the identification problem is amenable to solution via a quadratic optimization problem.

A.1 z is *measurable*. That is, there exists a LTI system $\mathcal{G}_m$ such that

$$\begin{bmatrix} \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} = \mathcal{G}_m \begin{bmatrix} \mathcal{L}_{ye} & \mathcal{L}_{yw} \end{bmatrix}.$$

This condition implies that z can be determined from measurements of y and u alone (along with knowledge of $\mathcal{L}$). This assumption is not as restrictive as might be feared at a first reading. Note that the spray deposition system, NARX model structure, and nonlinear circuit all satisfy condition A.1, as does the system of Fig. 1.

Our next major assumption concerns the experiment design problem of selecting an informative signal u. As our objective is to identify the nonlinear elements $\mathcal{N}$, it is more natural and direct to determine an optimal signal z, which is the input to $\mathcal{N}$. The following assumption allows us to determine an exogenous signal u that will generate z.

A.2 $\mathcal{N}$ and e are known, and z is *co-measurable*. That is, there exists a LTI system $\mathcal{G}_c$ such that

$$\begin{bmatrix} \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} = \mathcal{L}_{zu}\mathcal{G}_c.$$