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A new method for synthesizing multiple-period adaptive–repetitive controllers and its application to the control of hard disk drives ☆

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ABSTRACT

This paper introduces a new method for synthesizing multiple-period repetitive controllers. The main innovations in the synthesis procedure presented in this article are two. The first one is that this technique yields a solution compatible with the integration of the computed multiple-period repetitive controller into a minimum-variance adaptive control scheme. The second innovation is that the solution is period-recursive, reducing the complexity of controller synthesis considerably when compared with other methods available in the literature. To exemplify the synthesis procedure, a multiple-period repetitive controller is designed and integrated into an adaptive–repetitive control scheme used in the track-following control of a commercial hard disk drive. Experimental results show the effectiveness of the presented approach.

1. Introduction

Repetitive control (Hara, Yamamoto, Omata, & Nakano, 1998; Inoue, Nakano, Kubo, Matsumoto, & Baba, 1981; Tomizuka, Tsao, & Chew, 1989) has been demonstrated to be very effective in rejecting disturbances when implemented on systems affected by periodic disturbances, such as, *hard disk drives* (HDD), electric motors and generators, other rotating machines, and satellites (Broberg & Molyet, 1992a,b; Liang, Green, Weiss, & Zhong, 2002; Longman, Yeol, & Ryu, 2006; Pérez Arancibia, Lin, Tsao, & Gibson, 2007a; Senjyu, Miyazato, & Uezato, 1995; Yamada, Riadh, & Funahashi, 1999). Also, repetitive control has been shown to be an appropriate tool when applied to periodic tracking problems in power electronics, manufacturing and robotics (Cosner, Anwar, & Tomizuka, 1990; Costa-Castelló, Griñó, & Fossas, 2004; Ratcliffe, Hätönen, Lewin, Rogers, & Owens, 2006; Tsai, Anwar, & Tomizuka, 1988; Zhou & Wang, 2003; Zhou et al., 2007). In both kinds of problems, disturbance rejection and tracking, it is not rare to encounter applications where controllers capable of dealing with signals composed of multiple periods are required. Common examples are electromechanical systems containing multiple gears. This paper is devoted to the development of a new method for synthesizing repetitive controllers capable of rejecting multi-periodic output disturbances affecting the plant to be controlled.

The main feature of the method introduced here is that it produces multiple-period controllers suitable for integration into the combined adaptive–repetitive control scheme presented in Pérez Arancibia et al. (2007a), which is based on the notions of *internal model* (Francis & Wonham, 1976) and *adaptive minimum-variance regulation* (Horowitz, Li, & McCormick, 1998). The first part of this paper deals with the reformulation of the original disturbance rejection control problem as a polynomial algebraic one, and also, with finding an explicit analytical solution for it. In general, the existence of a solution with an explicit analytical expression does not guarantee simple computability. For this reason, the second part of this paper presents the development of a recursive algorithm that reduces significatively the complexity of control synthesis.

Previous works have addressed the problem of multiple-period repetitive control, from both theoretical and practical perspectives, (e.g., Garimella & Srinivasan, 1996; Krishnamoorthy & Tsao, 2005; Owens, Li, & Banks, 2004; Owens, Tomas-Rodriguez, Hätönen, & Li, 2006; Yamada et al., 1999; Yamada, Riadh, & Funahashi,

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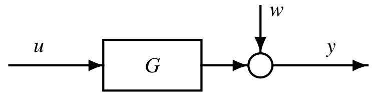

Fig. 1. LTI plant G and output disturbance w.

2000). However, those solutions are not easily integrable into the scheme presented in Pérez Arancibia et al. (2007a), considered here. For that reason, in this work we introduce an alternative approach, which extends the methods for designing one-period adaptive–repetitive controllers in Pérez Arancibia et al. (2007a) to the multi-periodic case, following the ideas and guidelines in Åström and Wittenmark (1984), Tomizuka (1987), Tomizuka et al. (1989) and Tsao and Tomizuka (1994). Experimental results obtained using a commercial HDD demonstrate the effectiveness of the resulting control synthesis method.

The rest of the paper is organized as follows. Section 2 reviews some fundamentals concepts of repetitive control. Section 3 presents the main contribution of this paper, which is a new method for synthesizing multiple-period repetitive controllers. Section 4 describes the multiple-period adaptive–repetitive control scheme to which the controller solution in Section 3 can be integrated to. Section 5 presents experimental results. Finally, some conclusions are given in Section 6.

Notation.

- z^{-1} denotes the delay operator, i.e., for a signal x, $z^{-1}x(t) = x(t-1)$ and conversely $zx(t) = x(t+1)$. Notice that since some of the systems involved in this paper are time-varying, here, z is not necessarily the complex variable associated to the z-transform.
- RH_∞ denotes the set containing all the LTI systems that are rational and stable as defined in Zhou, Doyle, and Glover (1996).
- $\|\cdot\|_2$ denotes the standard H_2 norm of a LTI system.
- $\|\cdot\|_\infty$ denotes the standard H_∞ norm of a LTI system.
- $|\cdot|$ denotes the standard module of a complex number.
- The upper index $^{(n)}$ is used to denote the recursion number n in a recursive algorithm. This does not denote an exponent.
- For a generic discrete random process $\mathbf{y}$, a realization of $\mathbf{y}$ is denoted by y.
- $\mathbb{N}$ denotes the set of positive integer numbers. $\mathbb{R}$ denotes the set of real numbers.

2. Preliminaries on repetitive control

2.1. Repetitive control for disturbance rejection

In this section, we review some fundamental ideas on one-period repetitive control that will be used later in this paper. First, consider the block diagram in Fig. 1. There, G is a stable LTI system and w is a disturbance considered to be mostly formed by a combination of sinusoidal sequences with frequencies multiple of a fundamental one. If the original plant system is unstable, it is assumed that it can be stabilized by LTI feedback control.

To begin with, we describe a repetitive control method for feedforward disturbance rejection in which the signal w is assumed to be available for measurement. This is a design assumption, since in practice w can be estimated but not directly measured. Also, it is assumed that the fundamental frequencies of the periodic signals forming part of w are a priori known. Thus, the natural control goal is the synthesis of a stable feedforward filter K, such that, the frequency response of the LTI system $1-KG$ is zero at the same periodic frequencies of the sinusoidal signals composing w. This approach results in the block diagram in Fig. 2, where

$$y = w - GKw = (1-GK)w. \quad (1)$$

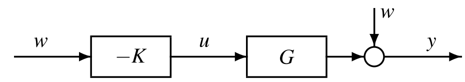

Fig. 2. Feedforward output disturbance rejection scheme.

Notice that the problem posed as in Fig. 2 becomes a feedforward tracking control problem. It is immediate that for the ideal case where G is minimum phase with relative degree 0, the best choice is to pick $K = G^{-1}$. However, it is not unusual to encounter discrete-time systems, obtained from sample-and-hold equivalence of continuous-time systems, that have unstable zeros. Thus, as in Tsao (1994), a possible design choice is to select a desired model M, and then find a minimizing K of some system norm of $M - GK$, for example, the H_∞ norm or the H_2 norm. Another option, the one chosen here as in Pérez Arancibia et al. (2007a), is to define an error transfer function $E = 1 - GK$ and then force the frequency response of E to be zero periodically at certain desired frequencies. This objective is achievable by using the polynomial design methods in Åström and Wittenmark (1984), following the general guidelines presented in Tomizuka et al. (1989) and Tomizuka (1987). The main idea is to enforce an error transfer function with the form $E = RD$, where D can be thought of as an internal model for the disturbance w, and R is an a priori unknown stable transfer function. For the one-periodic class of signals considered in this section, the internal model is chosen to be

$$D = 1 - qz^{-N}, \quad (2)$$

where q is a zero-phase low-pass filter and N is the period of the periodic disturbance to be attenuated.

The filter q will allow us some flexibility over the frequency range of disturbances to be canceled while maintaining stability. The filter D has a combed shape with notches matching the frequencies of the periodic signals forming part of w. Thus, a filter K that makes the frequency response of E zero at desired periodic frequencies can be computed by solving the Diophantine equation

$$RD + KG = 1, \quad (3)$$

where R and K are the unknowns.

Now, we briefly discuss the existence of solutions for (3). First, notice that (3) can be rearranged as

$$b_R\left(a_K a_G b_D\right) + b_K\left(a_R a_D b_G\right) = a_K a_G a_R a_D, \quad (4)$$

where the polynomial numerators are denoted by the symbol b, the polynomial denominators by the symbol a and the sub-indices indicate the corresponding transfer function in (3). It is immediate from Åström and Wittenmark (1984) and references therein, e.g., Kučera (1979), that for given polynomials a_G, b_D, a_D and b_G and chosen polynomials a_K and a_R, (4) has a solution if and only if the greatest common factor of $a_K a_G b_D$ and $a_R a_D b_G$ divides $a_K a_G a_R a_D$. In general if this condition is satisfied, we say that G and D are coprime.

As shown in Tomizuka et al. (1989) if a solution pair $\{R_o, K_o\}$ is found, then (3) characterizes a whole family of stabilizing internal model based repetitive controllers. As in Pérez Arancibia et al. (2007a), following the guidelines in Tomizuka (1987) and Tomizuka et al. (1989) a method for finding a particular solution pair $\{R_o, K_o\}$ is presented here. The general methodology of Tomizuka (1987) and Tomizuka et al. (1989) is also employed in Yamada et al. (2000) in the context of multiple-period repetitive control. The method starts by separating G into its minimum and non-minimum phase parts, denoted by G_+ and G_- respectively. Thus,

$$G = \frac{B}{A} = \frac{B_+ B_-}{A} = G_+ G_-, \qquad G_+ = \frac{B_+}{A}, \qquad G_- = B_-. \quad (5)$$