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Subspace-based prediction of linear time-varying stochastic systems☆

Kentaro Kameyama[a,*], Akira Ohsumi[b,1]

[a]Department of Mechanical Engineering, Fukui National College of Technology, Geshi, Sabae, Fukui 916 8507, Japan
[b]Department of Mechanical & System Engineering, Graduate School of Science and Technology, Kyoto Institute of Technology, Matsugasaki, Sakyo, Kyoto 606 8585, Japan

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Abstract

In this paper, a new subspace method for predicting time-varying stochastic systems is proposed. Using the concept of angle between past and present subspaces spanned by the extended observability matrices, the future signal subspace is predicted by rotating the present subspace in the geometrical sense, and time-varying system matrices are derived from the resultant signal subspace. Proposed algorithm is improved for fast-varying systems. Furthermore, recursive implementation of both algorithms is developed.

Keywords: Subspace prediction; Subspace methods; Subspace identification; Recursive algorithm; Time-varying systems

1. Introduction

Subspace-based system identification method which was proposed in the early 1970s has been attracting much attention because of its affinity to the modern control theory which is based on the state space model. Since then, many researches have been made in this area, achieving a significant level of maturity and acceptability in control system applications. For instance, refer Verhaegen and Dewilde (1992a, 1992b), Van Overschee and De Moor (1993, 1994, 1995, 1996a), Verhaegen (1994), Viberg (1995), and Ohsumi, Takashima, and Kameyama (1997) for discrete-time systems, and also Bastogne, Garnier, Sibille, and Mensler (1996), Haverkamp, Chou, Verhaegen, and Johansson (1996), Haverkamp, Verhaegen, Chou, and Johansson (1997), Van Overschee and De Moor (1996b), Huang and Katayama (2001), and Ohsumi, Kameyama, and Yamaguchi (2002) for continuous-time systems. Nowadays, it is well recognized that subspace identification is very efficient to model multivariable linear time-invariant systems and successfully applied to peculiar industrial systems with high-order and multiple inputs and outputs (e.g., Favoreel, De Moor, & Overschee, 2000).

In order to obtain mathematical models for accurate control, the estimation and/or prediction of the system change are significantly important, especially, in order to realize the model predictive control (Camacho & Bordons, 1999; Richalet, 1993; Maciejowski, 1999). The model predictive control has succeeded in industrial and practical process control applications in the last two decades. It seems to be one of the most attractive topics in the field of process control engineering. Industrial experience shows that the most difficult and time-consuming work in the model predictive control is modeling and identification. Widespread application of such technology calls for more effective and efficient methods of multivariable process identification. This suggests necessarily that a happy *mariage* between subspace system identification and model predictive control is welcome. From such a standpoint a method was presented by Favoreel, De Moor, and Gevers (1999) to combine the subspace identification and the predictive control design of linear systems in one single operation.

Keeping in mind the future application to model predictive control, the authors investigate in this paper the possibility of

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* Corresponding author. Tel.: +81 778 62 8315; fax: +81 778 62 3306.
E-mail addresses: k_kame@fukui-nct.ac.jp (K. Kameyama), ohsumi@cs.miyazaki-u.ac.jp (A. Ohsumi).
[1] Professor Emeritus of Kyoto Institute of Technology.
Present address: Department of Computer Science and System Engineering, Faculty of Engineering, University of Miyazaki, Kibana, Gakuen, Miyazaki 8892192, Japan.

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predicting unknown system from the subspace identification point of view. The key to approach to the subspace-based prediction (SP) of the unknown system is to introduce the idea of angle between two subspaces, past and present subspaces, or present and future subspaces. The implementation of the SP algorithm is realized using a recursive algorithm in order to save computational burden and storage costs.

The paper is organized as follows. Section 2 provides the problem formulation. Some mathematical preliminaries concerning to the angle between subspaces and the rotation of subspace are provided in Section 3. SP algorithms are stated in Sections 4 and 5. In Section 6 a recursive form of the SP algorithm is given. Illustrative examples are provided in Section 7. Conclusions are stated in Section 8.

2. Problem statement

Suppose that we are given a couple of input and output data sequences $\{u_k, y_k\}_{k=1,2,\ldots}$ and that the output data is generated from the discrete-time time-varying stochastic system:

$$\begin{cases} x_{k+1} = A_k x_k + B_k u_k + w_k, & (1) \\ y_k = C_k x_k + D_k u_k + v_k, & (2) \end{cases}$$

where $u_k \in R^m$, $y_k \in R^\ell$, and $x_k \in R^n$ are input, output and state vectors; $w_k \in R^n$ and $v_k \in R^\ell$ are zero-mean white Gaussian sequences with covariance matrix:

$$\mathscr{E}\left\{\begin{bmatrix} w_k \\ v_k \end{bmatrix}[\, w_s^{\mathrm T} \quad v_s^{\mathrm T}\,]\right\} = \begin{bmatrix} Q & S \\ S^{\mathrm T} & R \end{bmatrix}\delta_{ks}, \tag{3}$$

where δ_{ks} is the Kronecker delta.

Our problem is to identify and predict the time-varying quadruple system matrices ($A_{k+\mu}$, $B_{k+\mu}$, $C_{k+\mu}$, $D_{k+\mu}$) at μ-step ahead (within a similarity transformation) from a given set of input and output data of the unknown linear time-varying system (1)–(2), including the system order n, up to the present time step k.

In this paper, "time-varying" means that all system matrices as well as noise covariance matrices change slowly with time. Qualitatively speaking, the instinctive word "slowly" implies that all matrices change smoothly and continuously, and do never abruptly or randomly. Then, under the assumption that system can be regarded as time-invariant in the fixed time-interval if system changes slowly with time (Ohsumi, Matsuüra, & Kameyama, 2003; Ohsumi & Kawano, 2002), the input–output algebraic relationship with argument k is given as

$$\begin{aligned} Y_\alpha(k\,|\,k-N+1) = {} & \Gamma_\alpha(k) X_\alpha(k\,|\,k-N+1) \\ & + H_\alpha(k) U_\alpha(k\,|\,k-N+1) \\ & + \Sigma_\alpha(k) W_\alpha(k\,|k-N+1) \\ & + V_\alpha(k\,|\,k-N+1), \end{aligned} \tag{4}$$

where $Y_\alpha(k\,|\,k-N+1) \in \mathbf{R}^{\alpha\ell\times N}$ is a block Hankel matrix:

$$Y_\alpha(k\,|\,k-N+1) = \begin{bmatrix} y_{k-N-\alpha+2} & y_{k-N-\alpha+3} & \cdots & y_{k-\alpha+1} \\ y_{k-N-\alpha+3} & y_{k-N-\alpha+4} & \cdots & y_{k-\alpha+2} \\ \vdots & \vdots & & \vdots \\ y_{k-N+1} & y_{k-N+2} & \cdots & y_k \end{bmatrix},$$

$U_\alpha(\cdot\,|\,\cdot) \in \mathbf{R}^{\alpha m\times N}$, $W_\alpha(\cdot\,|\,\cdot) \in \mathbf{R}^{\alpha n\times N}$, and $V_\alpha(\cdot\,|\,\cdot) \in \mathbf{R}^{\alpha\ell\times N}$ are identical to $Y_\alpha(\cdot\,|\,\cdot)$ except u_k, w_k, and v_k instead of y_k, respectively; $X_\alpha(\cdot|\cdot) \in \mathbf{R}^{n\times N}$ is the matrix constructed by system states:

$$X_\alpha = [\, x_{k-N-\alpha+2} \quad x_{k-N-\alpha+3} \quad \cdots \quad x_{k-\alpha+1}\,],$$

$\Gamma_\alpha(\cdot) \in \mathbf{R}^{\alpha\ell\times n}$ is the extended observability matrix; $H_\alpha(\cdot) \in \mathbf{R}^{\alpha\ell\times\alpha\ell}$ and $\Sigma_\alpha(\cdot) \in \mathbf{R}^{\alpha\ell\times\alpha n}$ are lower block triangular matrices consisting of system matrices $\{A_k, B_k, C_k, D_k\}$ (Ohsumi & Kawano, 2002).

Here, we should notice that unknown system quadruplet at time step k is derived by using the $\mathrm{span_{col}}\{\Gamma_\alpha(k)\}$ in the 4SID methods, where the notation $\mathrm{span_{col}}\{\Gamma_\alpha(k)\}$ means the vector subspace spanned by the column vectors of $\Gamma_\alpha(k)$ and is called the signal subspace at time step k. Hence, our problem is just to predict the signal subspace at a future step $k+\mu$. We call the problem a subspace prediction (SP) problem. To solve this problem, we introduce the concept "angle between two subspaces" which relates geometrically the two given signal subspaces at different time steps. Using this concept, the change of signal subspace due to time-varying system parameters can be represented by its rotational evolution. Thus, the future signal subspace can be predicted by rotating the present signal subspace. So, the SP problem is to predict the signal subspace at the future time step $k+\mu$ from a given set of input and output data of the unknown linear time-varying system (1)–(2) up to the present time step k. The quadruple system matrices ($A_{k+\mu}$, $B_{k+\mu}$, $C_{k+\mu}$, $D_{k+\mu}$) at μ-step ahead are derived from the resultant future signal subspace in the sense of least squares (Van Overschee & De Moor, 1996a; Verhaegen, 1994). In this paper, two types of SP algorithms will be proposed.

3. Mathematical preliminaries

3.1. Angle between subspaces

Consider two matrices $A \in R^{p\times r}$ and $B \in R^{q\times r}$ $(p, q \leqslant r)$ with rank $A = a_d$ and rank $B = b_d$, respectively. Then, the angle between two subspaces spanned by column vectors of A and B is defined by a set of angles $\{\theta_i,\ i = 1, 2, \ldots, a_d \wedge b_d\}$ (where $a_d \wedge b_d = \min(a_d, b_d)$, $0 \leqslant \theta_i \leqslant \pi/2$) between principal vectors $a_i \in \mathrm{span_{col}}\{A\}$ and $b_j \in \mathrm{span_{col}}\{B\}$ $(i, j = 1, 2, \ldots, a_d \wedge b_d)$. The following is the definition of angle between subspaces (e.g., Golub & Van Loan, 1989; Van Overschee & De Moor, 1996a).

Definition. Given two matrices A and B mentioned above, choose first a pair of principal vectors a_1 and b_1 among the