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Validating the J-value safety assessment tool against pan-national data

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ABSTRACT

The J-value is an objective method for determining when life extending measures are sensible, applicable to both manufacturing and service industries, including public health and healthcare. A model of human decision making based on the J-value is able to explain the shape of the Preston curve that relates life expectancy at birth and gross domestic product (GDP) per head for all the nations in the world. Making a number of reasonable assumptions, a J-value model produces a population-average life expectancy, which may be translated easily into a corresponding life expectancy at birth when life expectancy is not modified by discounting (net discount rate equals zero). The resultant values may be tested against pan-national data, showing a very good match. Thus the shape of the Preston curve has been explained and, at the same time, validation has been provided for the J-value model. A perturbation analysis shows that the J-value explanation for the Preston curve starts to break down as the net discount rate is increased above zero. Thus the Preston curve may be seen to validate the J-value model at a net discount rate of zero, but not at higher net discount rates. The result allows a closed-form expression to be derived for the first time for the pure time discount rate, namely the product of the rate of economic growth and the complement of risk-aversion. A further conclusion from the work is that no discernable limit is apparent before the age of 100 to the process by which people live longer as they get richer; such an intrinsic limit might be overcome by future improved medical technology.

1. Introduction

The J-value (Thomas et al., 2006a,b; 2010a) is an objective method for determining when life extending measures are sensible, applicable to both manufacturing and service industries. Based on the life-quality index (Nathwani and Lind, 1997; Nathwani et al., 2009), the method assumes that a rational trade-off is made between an increase in life expectancy and the cost of the measure that brings about that increase, with the ultimate objective of maintaining or improving life quality.

The J-value has the considerable advantage over conventional cost benefit analysis that no explicit assumptions have to be made about the difficult issue of the monetary value to be attached to saving a human life. Instead of using subjective stated preferences of a small sample of the population exposed to a hazard, it is instead grounded in objective actuarial and economic statistics characterising the lives and behaviours of millions of citizens. It is thus suitable for assessing health and safety measures across all industries, from oil and gas, chemical and nuclear through transport to the National Health Service in the UK. Also, unlike other approaches, the J-value allows immediate fatalities and loss of life in the longer term (e.g. after exposure to a carcinogen) to be measured on the same scale. It is thus particularly suitable for judging nuclear safety, including assessing mitigation strategies following large accidents like Chernobyl and Fukushima Daiichi.

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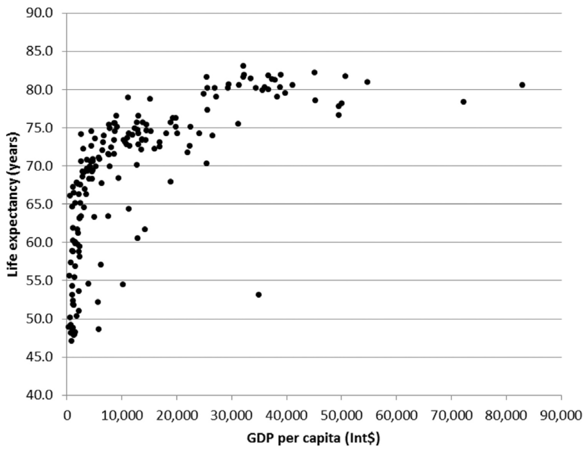

Fig. 1 – The Preston curve—life-expectancy at birth (years) as a function of GDP per capita (international dollars) for the 180 countries for which both datasets are available for 2009 (OECD, 2011; World Bank, 2012a,b).

An ethical principle of J-value analysis is that the next day of life should be valued the same for everyone in the nation, old or young, rich or poor. This principle is reflected in the use of the gross domestic product (GDP) per head as the baseline annual income used in the definition of life quality.

The Preston curve (Preston, 1975) highlights the fact that there is a clear, positive correlation between the GDP per head in different countries in the world and the average length of time people in those countries can expect to live. See Fig. 1, which shows results from 180 out of the 193 nations affiliated to the United Nations. To the authors' knowledge, no successful, theoretical explanation has been put forward for the shape of the Preston curve up to this point.

The paper will suggest how the actions of the peoples in different nations in the world can be characterised by a life-extension assessment procedure based on J-value principles, and the resultant mathematical model will be used to explain the form of the Preston curve. The process of explanation can also be seen as a test of the J-value model, and, indeed a severe test, since the model is required to match data derived from the decisions of almost everyone in the world when grouped together into nations. Hence passing this test will provide a substantial degree of validation (Popper, 1934; Butterfield and Thomas, 1986a,b; Thomas, 1999) for the J-value method. This validation exercise is additional to and complementary to the corroboration reported in Thomas (2017a,b), where the J-value model was tested against UK data on life expectancy at birth.

2. The J-value

The J-value is derived from the life quality index (LQI), Q, (Nathwani and Lind, 1997; Pandey et al., 2006; Nathwani et al., 2009; Thomas et al. 2006a, 2010a):

$$Q = G^{1-\varepsilon} X_d \tag{1}$$

where G is the income per person, taken for ethical reasons to be the GDP per head (£/year) and thus the same for everyone in the same national jurisdiction, while ε is the risk-aversion associated with measures that will extend life expectancy, estimated previously at between 0.82 and 0.85 for the UK (Thomas et al., 2010a,b). See also Blundell et al. (1994) who suggest a figure of 0.83 using a diverse method and Pearce and Ulph (1995), who suggest a range 0.8–0.9. Meanwhile X_d is the discounted life expectancy of the population as a whole (years).

It may be seen from Eq. (45) of Thomas et al. (2010a), that discounting is actually applied to the utility of income in future years, but the fact that the necessary integrand consists of the product of discount factor, utility and survival probability means that it is convenient to define the integral of {survival probability × discount factor} as the "discounted life expectancy". The choice of title reflects the fact that the discounted life expectancy, X_d, will degenerate to the life expectancy, X, when the net discount rate is zero. The net discount rate, r, is given generally by

$$r = \lambda + g\varepsilon - g = \lambda - g(1-\varepsilon) \tag{2}$$

where λ is the pure time preference rate and g is the growth rate of the economy, both of which might differ between different nations (Thomas, 2012).

A condition for a life-extending measure to be rationally justified is that the life quality index should not fall as a result of a person spending an annual amount, δG, from his income on the health and safety measure for the rest of his expected life, causing a decrement, $-\delta G$ in his annual income. In line with the Kaldor–Hicks compensation principle (Kaldor, 1939; Hicks, 1939), while the individual would be prepared to spend such an amount, the annual payment might actually be made (and in many cases will be made) by some other person or body. Assuming a constant value of net discount rate, r, the change in LQI due to small changes in income and discounted life expectancy, δX_d, is

$$\begin{aligned}\delta Q &= -\frac{\partial Q}{\partial G}\delta G + \frac{\partial Q}{\partial X_d}\delta X_d \\ &= -(1-\varepsilon)G^{-\varepsilon}X_d\delta G + G^{1-\varepsilon}\delta X_d\end{aligned} \tag{3}$$

Dividing by Eq. (1), we find

$$\frac{\delta Q}{Q} = -(1-\varepsilon)\frac{\delta G}{G} + \frac{\delta X_d}{X_d} \tag{4}$$

The maximum rational annual expenditure on life extension will occur when $\delta Q = 0$ and, for non-zero Q, this will occur when

$$\frac{\delta X_d}{X_d} = (1-\varepsilon)\frac{\delta G}{G} \tag{5}$$

which thus defines δG. If the actual annual expenditure is $\delta\hat{G}$, then the J-value is given by:

$$J = \frac{\delta\hat{G}}{\delta G} \tag{6}$$

Thus $J = 1$ defines the locus of a curve in the plane of G versus X_d where life quality, Q, is maintained constant. See Fig. 2.

3. Life expectancy and discounted life expectancy

The life expectancy at age a is given by the integral of the conditional survival probabilities:

$$X(a) = \int_{t=a}^{\infty} S(t|a)\,dt == \int_{t=a}^{\infty} \frac{S(t)}{S(a)}dt \tag{7}$$

where $S(t)$ is the cumulative probability of survival to age, t, from age, 0. It may be helpful, by way of example, to observe that the life expectancy at birth, $X(0)$, averaged across the two