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Nonlinear minimum-time control with pre- and post-actuation☆

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ABSTRACT

This article studies the time-optimal output transition problem to change the system output, from an initial value $y(t) = \underline{y}$ (for all time $t \leq 0$) to a final value $y(t) = \bar{y}$ (for all time $t \geq T$), for invertible nonlinear systems. The main contribution of the article is to show that the use of pre- and post-actuation input outside the transition interval $\mathbb{I}_T = [0, T]$ can reduce the transition time T beyond the standard bang–bang-type inputs for optimal state transition. The advantage of using pre- and post-actuation is demonstrated with an illustrative nonlinear example.

1. Introduction

The minimum-time state transition problem with bounds on the input magnitude leads to the classical bang–bang-type input for the fastest state transition. However, the transition time can be reduced further if only the system output needs to be transitioned from one value to another rather than the entire system state. The time-optimal output transition problem is to change the system output from an initial value $y(t) = \underline{y}$ (for all time $t \leq 0$) to a final value $y(t) = \bar{y}$ (for all time $t \geq T$), as shown in Fig. 1. The contributions of this article are (i) to quantify the additional flexibility with output transition (as opposed to standard state transition from an initial value $x(0)$ to a final value $x(T)$) in terms of geometric conditions on the internal dynamics of the system and (ii) to show that the use of pre- and post-actuation input outside the transition interval $\mathbb{I}_T = [0, T]$ can reduce the transition time T beyond the standard bang–bang-type inputs for optimal state transition. The advantage of using pre- and post-actuation is demonstrated with an illustrative nonlinear example.

As opposed to the optimal output transition (OOT) problem, the time-optimal transition between two states has been well studied in literature (Boscain & Piccoli, 2001; Bryson & Ho, 1975; Frank & Vassilis, 1995; Sussmann, 1990; Wing & Desoer, 1963). Such time-optimal solutions to the standard state transition (SST) from $x(0) = \underline{x}$ to $x(T) = \bar{x}$ can be used to solve the output transition problem. For example, by choosing the boundary states $\underline{x}, \bar{x}$ to be equilibrium states (corresponding to the boundary values of the output $\underline{y}, \bar{y}$), SST approaches can reduce the time needed for the output transition. This choice of boundary states ensures that the output remains at the desired (constant) values outside the output-transition time interval $\mathbb{I}_T$ without the need for pre- and post-actuation. Such time-optimal state transition problems have been studied for a variety of applications such as: (i) endpoint positioning of large-scale manipulators (Farrenkopf, 1979; Singhose, Banerjee, & Seering, 1997); (ii) maneuvering of spacecraft (Ben-Asher, Burns, & Cliff, 1992; Singh, Kabamba, & McClamroch, 1989; Thompson, Junkins, & Vadali, 1989; Tzes & Yurkovich, 1993); (iii) maneuvering of flexible structures (Chen & Desrochers, 1990; Hindle & Singh, 2001; Meckl & Kinceler, 1994; Pao & Frankin, 1990); (iv) positioning of read-write heads in disk drives (Hai, 1997; La-orpacharapan & Pao, 2004; McCormick & Horowitz, 1991) and (v) nano-scale positioning using relatively-smaller piezoactuators (Moallem, Kermani, Patel, & Ostojic, 2004). In the nonlinear setting, computational issues for the state transition problem are studied in Kim, Choi, and Ha (2005). Moreover, previous investigations include the synthesis of feedback-type solutions for the optimal control problem (Boscain & Piccoli, 2001; Sussmann, 1987, 1990), the minimum-time swing-up for the pendulum-and-cart problem (Mason, Broucke, & Piccoli, 2008), as well as the minimal-time transition between different cart positions for the pendulum-and-cart problem (Matthew & Raffaello, 2005). While the SST approach reduces the output transition time T, the present article shows that the minimum transition time with the SST approach can be reduced further by directly solving the minimum-time OOT problem.

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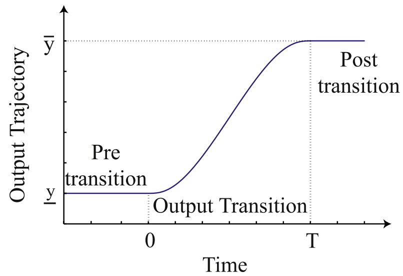

Fig. 1. Minimum-time output transition problem: the output trajectory has to be maintained at a constant initial value $\underline{y}$ before the beginning of the output-transition ($y(t) = \underline{y}, t \leq 0$), and at a constant final value $\bar{y}$ after the completion of the output-transition ($y(t) = \bar{y}, t \geq T$).

The problem of minimizing the time T needed to reach a desired output $y(T) = \bar{y}$ for linear systems was studied in Lewis (1981). A nonlinear version of the problem of changing part of the state (say the output) to a desired value in minimum time has been recently studied in Chang, Petit, and Rouchon (2006). As opposed to achieving the desired output at a particular time instant (as in Chang et al., 2006; Lewis, 1981), the issue of also maintaining the output afterwards (i.e., $t \geq T$) at the desired value $\bar{y}$ was studied in Emami-Naeini (1992) for linear systems. Computational challenges in the nonlinear extensions of this output transition problem are addressed in Nesic and Mareels (1998). The current work extends previous work on the nonlinear output transition problem to include both pre- and post-actuation. In particular, the proposed approach allows the system to evolve in the internal dynamics (outside of the transition interval $\mathbb{I}_T$). Thus, it ensures that the output is kept at the desired values by using pre-actuation input (for $t \leq 0$) and post-actuation input (for $t \geq T$). Note that the use of pre- and post-actuation effectively increases the time available to apply the input, without an increase in the time T needed for the output transition. The resulting availability of additional time to apply inputs (during pre- and post-actuation) tends to lower the required output-transition time T.

The problem of optimal output transition for linear systems, with pre- and post-actuation, was posed in Dowd and Thanos (2000), Piazzi and Visioli (2000, 2001), which find the minimum-time solution from a pre-specified class of trajectories. For example, polynomials were used to pre-specify a set of output trajectories from which a minimal-time solution was obtained in Piazzi and Visioli (2000). However, the output and input trajectories are not intuitive for solutions to typical minimum-time problems, and therefore, it is challenging to include them in the initial set of pre-specified trajectories. In contrast to choosing the output trajectory from a pre-specified set, the input and output trajectories are found as part of the optimization process with the minimum-time OOT approach. For the linear case, previous works have shown that the use of pre- and post-actuation can be used to reduce cost functions involving the input energy (Iamratanakul, Jordan, Leang, & Devasia, 2008; Perez & Devasia, 2003). The current article extends such pre- and post-actuation in two aspects: (a) minimization of the transition time rather than a cost function that involves the input energy, and (b) generalization to the nonlinear setting. Although the current approach is not formulated to handle output constraints such as reducing overshoot in minimum-time problems (the handling of such output constraints was studied recently in Consolini and Piazzi (2009)), the additional flexibility with the potential use of pre- and post-actuation could lead to the reduction of the transition time even with such output constraints.

2. Problem formulation

The output transition problem is posed for invertible nonlinear systems.

2.1. System description

Consider a system described by

$$\begin{aligned} \dot{x}(t) &= f[x(t)] + g[x(t)]u(t) \\ y(t) &= h[x(t)] \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) = [y_1(t), y_2(t), \ldots, y_p(t)]^T$ is the output, with the same number of inputs as outputs, i.e., $u(t), y(t) \in \mathbb{R}^p$, and the input is bounded as

$$\|u(t)\|_\infty \leq U_{\max} \quad \forall -\infty < t < \infty. \tag{2}$$

2.2. Optimal output transition (OOT) problem

Let $\underline{x}$ and $\bar{x}$ be controlled equilibrium points of the system (Eq. (1)) corresponding to inputs $\underline{u}$ and $\bar{u}$ and outputs $\underline{y}$ and $\bar{y}$, i.e.,

Definition 1 (*Delimiting States for Transition*).

$$\begin{aligned} f[\underline{x}] + g[\underline{x}]\underline{u} = 0, &\quad \underline{y} = h[\underline{x}] \\ f[\bar{x}] + g[\bar{x}]\bar{u} = 0, &\quad \bar{y} = h[\bar{x}]. \end{aligned} \tag{3}$$

The output transition problem is formally stated next.

Definition 2 (*Output Transition Problem*). Given the delimiting states and a transition time interval $[0, T]$, find a bounded input-state trajectory $[u_{ref}(\cdot), x_{ref}(\cdot)]$ that satisfy the system equations (1), (2) for all time ($-\infty < t < \infty$)

$$\begin{aligned} \dot{x}_{ref}(t) &= f[x_{ref}(t)] + g[x_{ref}(t)]u_{ref}(t) \\ y_{ref}(t) &= h[x_{ref}(t)] \\ \|u_{ref}(t)\|_\infty &\leq U_{\max} \end{aligned} \tag{4}$$

and the following two conditions.

[I. *The output transition condition*] The output transitions in the time interval $\mathbb{I}_T = [0, T]$ and is maintained at the desired value outside the time interval $\mathbb{I}_T$, i.e.,

$$\begin{aligned} &\text{from } y_{ref}(t) = \underline{y} && \text{for all time } t \leq 0 \\ &\text{to } y_{ref}(t) = \bar{y} && \text{for all time } t \geq T. \end{aligned} \tag{5}$$

[II. *The delimiting state condition*] The system state approaches the delimiting states as time goes to (plus or minus) infinity,

$$\begin{aligned} x(t) &\to \underline{x} \quad \text{as } t \to -\infty \\ x(t) &\to \bar{x} \quad \text{as } t \to \infty. \end{aligned} \tag{6}$$

The time-optimal output transition seeks to minimize the transition time T with constraints on the input.

Definition 3 (*OOT*). The minimum-time optimal output transition (OOT) problem is to find the bounded input-state trajectory $[u^*(\cdot), x^*(\cdot)]$ that satisfies the output transition problem (in Definition 2), and minimizes the transition time T with the cost function

$$J = \int_0^T 1 \, dt = T. \tag{7}$$

3. Solution

The section begins with the standard approach based on time-optimal state transition, followed by the solution to the time-optimal output transition problem for invertible systems.