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An adaptive method for consistent estimation of real-valued non-minimum phase zeros in stable LTI systems[☆]

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ABSTRACT

An adaptive algorithm, consisting of a recursive estimator for a finite impulse response model having two non-zero lags only, and an adaptive input are presented. The model is parametrized in terms of the first impulse response coefficient and the model zero. For linear time-invariant single-input single-output systems with real rational transfer functions possessing at least one real-valued non-minimum phase zero of multiplicity one, it is shown that the model zero converges to such a zero of the true system. In the case of multiple non-minimum phase zeros, the algorithm can be tailored to converge to a particular zero. The result is shown to hold for systems and noise spectra of arbitrary degree. The algorithm requires prior knowledge of the sign of the high frequency gain of the system as well as an interval to which the non-minimum phase zero of interest belongs.

1. Introduction

How to cope with system complexity is a key issue in system identification. A model is always an idealization of the real world and the purpose of modeling is to capture characteristics of the real system behavior that are important for the application at hand, despite that the complete system behavior cannot be modeled. It has long been recognized that the meaningful modeling should be based on the intended model use; see for example Gevers and Ljung (1986). This issue was very much brought into focus during the efforts, initiated in the early 1990s, to address the so-called identification for control problem (Gevers, 1993; Goodwin, Gevers, & Ninness, 1992; Rivera & Jun, 2000; Van den Hof & Schrama, 1995). A key outcome here was the recognition of the importance of the experiment design[1] and this leads to iterative approaches trying to achieve experimental conditions such that the bias error was distributed over frequencies to suit control applications. However, also computational methods for optimal experiment design were revisited and extended (Hildebrand & Gevers, 2003; Jansson & Hjalmarsson, 2005; Lindqvist & Hjalmarsson, 2001).

A well-known problem in system identification is the curse of complexity, i.e. the model uncertainty grows with the system complexity so that for highly complex systems the model becomes virtually useless. Through some simple examples, it was advocated in Hjalmarsson (2005) that it is possible to combat this problem by careful experiment design and that this also allows simple models to be used (as long as only a limited amount of system properties are to be extracted from the measurements). The cost of an experiment vs. the amount of information to be extracted has been formalized in Rojas, Syberg, Welsh, and Hjalmarsson (2010) for frequency function estimation and later on elaborated upon in Hjalmarsson (2009) for general system properties.

Following up on Hjalmarsson (2005), the dual role of a "good" input as (1) an enhancer of system properties of interest, and (2) as an attenuator of properties of little or no interest was formalized in Mårtensson and Hjalmarsson (2011). In particular it was shown that, under certain conditions, an input that is designed to be optimal for a scalar cost function and for a full order model, results in experimental data for which also reduced order models can be used to consistently identify the property of interest.

The special case of identification of real non-minimum phase (NMP) system zeros has been considered in Jansson (2004); Mårtensson, Jansson, and Hjalmarsson (2005) where it is shown that the input

$$u_{n+1} = z_*^{-1} u_n + r_n, \qquad (1)$$

where z_* is the NMP zero of interest and $\{r_n\}$ is white noise, allows z_* to be consistently identified using a two parameter FIR model.

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[1] This, of course, was well known earlier on also but during this time its importance became very palpable.

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Even though conceptually interesting, the catch of this result from a practical point of view is, of course, that the input depends on the zero to be identified. Hence, even though it is shown in Jansson (2004) that the choice of the pole of the input spectrum is not so critical for the accuracy of the estimate, the practical applicability of the result is limited. It is common that an optimal experiment design depends on the true system and there are two main approaches to circumvent this problem: (1) robust input design where the design takes into account that the true system lies in an uncertainty set a priori (Rojas, 2008), and (2) adaptive, or sequential, design where the design is successively updated as new data is collected from the system (Lindqvist & Hjalmarsson, 2001). Recently, it has been shown (Gerencsér, Hjalmarsson, & Mårtensson, 2009) that when the true system is in the model set and when an ARX model is used, adaptive input design achieves asymptotically (in the sample size) the same accuracy as a non-adaptive design where the true system is known. See also the recent survey Pronzato (2008).

In this contribution we revisit the zero estimation problem and propose an adaptive approach. We consider stable rational causal discrete-time linear time-invariant systems subject to stationary stochastic disturbances. We show that it is possible to estimate a real-valued zero of multiplicity one outside the unit circle consistently using a simple two parameter FIR model, if the input can be manipulated and some prior information regarding the location of the zero of interest is available. The main contribution is the convergence analysis of this algorithm. We remark that existing consistency results of recursive identification/adaptive control algorithms in the case of severe undermodeling, which is the case in this contribution, are very limited.

Our motivation for this study is two-fold. Firstly, NMP zeros are important in control applications as they limit closed loop performance (Skogestad & Postlethwaite, 1996) and in many applications such zeros are real-valued and simple, e.g. drum boiler dynamics (Kwatny & Berg, 1993) and aircraft dynamics (Dahleh & Diaz-Bobillo, 1995). Secondly, the standard assumption that the true system belongs to the model set is unrealistic in many applications. Our contribution suggests a method to ensure that important system quantities can be estimated consistently despite model limitations. To examine the viability of this path is left for future research.

The outline is as follows. Assumptions are introduced in Section 2. Consistency for models of restricted complexity is discussed in Section 3. An adaptive algorithm is proposed in Section 4 and the ODE (ordinary differential equation) corresponding to this algorithm is analyzed in Section 5. The main convergence result is provided in Section 6 and the result is illustrated in Section 7 by way of a simulation example.

2. Assumptions

The assumptions in this section are assumed to hold throughout the entire paper.

Assumption 2.1 (*System*)**.** The system has a state-space representation of the form

$$\begin{aligned} \xi_{n+1} &= A^o\xi_n + B^o u_n + K^o e_n^o \\ y_n &= C^o\xi_n + e_n^o \end{aligned} \tag{2}$$

where $u_n \in \mathbb{R}$ and $y_n \in \mathbb{R}$ represent the input and measured output at time n, respectively, where $\xi_n \in \mathbb{R}^m$, for some positive integer m, is the state vector, and where $e_n^o \in \mathbb{R}$ represents noise acting on the system.

The transition matrix A^o has all its eigenvalues strictly inside the unit circle, i.e. the system is internally stable.

The input–output relationship of the system is given by

$$y_n = G^o(q)u_n + w_n^o, \tag{3}$$

where

$$G^o(q) = C^o(qI - A^o)^{-1}B^o = \sum_{k=1}^{\infty} g_k^* q^{-k} \tag{4}$$

and $w_n^o = H^o(q)e_n^o$ with $H^o(q) = C^o(qI - A^o)^{-1}K^o + 1$. The system has one pure time-delay, i.e. $g_1^* \neq 0$.

The system has a real-valued NMP zero of multiplicity 1 at an unknown location z_*, i.e. $G^o(z_*) = 0$ where $z_* \in \mathbb{R}$, $|z_*| > 1$.

Notice that the system may have other zeros than z_*, real as well as complex valued.

Assumption 2.2 (*Prior System Knowledge*)**.** The following prior knowledge is assumed:

(i) A real compact set

$$\mathcal{G} = \{g_1 : \underline{g}_1 \le g_1 \le \bar{g}_1\}, \quad 0 \notin \mathcal{G},\ g_1^* \in \mathcal{G}. \tag{5}$$

(ii) A compact interval $\mathcal{Z} \subset \mathbb{R}$ with the following properties

$$\begin{aligned} &G^o(z) = 0, \quad z \in \mathcal{Z} \Rightarrow z = z_* \\ &z \in \mathcal{Z} \Rightarrow |z| > 1. \end{aligned} \tag{6}$$

(iii) The parity of the number of system zeros on the ray $\{\alpha z_*, \alpha > 1\}$ is known.

Notice that since the interval defining $\mathcal{G}$ can be arbitrarily large, the assumption that a set $\mathcal{G}$ is known effectively means that the sign of the first impulse response coefficient has to be known. This is equivalent to the assumption of knowledge of the sign of the high frequency (or instantaneous) gain frequently used in adaptive control (Åström & Wittenmark, 1995).

Assumption 2.3 (*Noise*)**.** The noise $\{e_n^o\}$ is a sequence of independent random variables of zero mean and variance λ_o for which

$$\sup_n \mathbf{E}[e^{\varepsilon (e_n^o)^2}] < \infty, \tag{7}$$

holds for some $\varepsilon > 0$.

We remark that (7) holds for bounded random variables as well as for Gaussian random variables.

Assumption 2.4 (*Input*)**.** The input is generated by

$$u_{n+1} = \rho_n^{-1}u_n + \sqrt{\lambda_u}\sqrt{1 - \rho_n^{-2}}\,r_n \tag{8}$$

where $\lambda_u > 0$ is a user-defined constant and where $\{r_n\}$ is a sequence of independent random variables of zero mean and unit variance. Furthermore, $\{r_n\}$ is independent of $\{e_n^o\}$ and subject to the condition

$$\sup_n \mathbf{E}[e^{\varepsilon (r_n)^2}] < \infty \quad \text{for some } \varepsilon > 0. \tag{9}$$

The motivation for (8) stems from (1). A recursive estimate of z_* will be used for the sequence $\{\rho_n\}$. This is part of the algorithm and will be discussed later. The factor $\sqrt{\lambda_u}\sqrt{1 - \rho_n^{-2}}$ in (8) ensures that the power $\mathbf{E}[u_n^2]$ of the input equals λ_u when $\rho_n = \rho \in \mathbb{R}$ (constant), with $|\rho| > 1$.

3. Consistency and models of restricted complexity

3.1. Introduction

Consistency is one of the key issues in system identification. This concerns whether the system will be recovered by the estimated model as the number of data samples grows unbounded.