Automatica 47 (2011) 1399–1408

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Multisensor fusion for linear control systems with asynchronous, Out-Of-Sequence and erroneous data[☆]

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ARTICLE INFO

Article history:
Received 22 July 2009
Received in revised form
23 December 2010
Accepted 23 January 2011
Available online 17 March 2011

Keywords:
Estimation theory
Information and sensor fusion
Systems with time-delays
Sensor networks
Kalman filtering
Out-of-sequence data

ABSTRACT

This paper presents a set of new centralized algorithms for estimating the state of linear dynamic Multiple-Input Multiple-Output (MIMO) control systems with asynchronous, non-systematically delayed and corrupted measurements provided by a set of sensors. The delays, which make the data available Out-Of-Sequence (OOS), appear when using physically distributed sensors, communication networks and pre-processing algorithms. The potentially corrupted measurements can be generated by malfunctioning sensors or communication errors. Our algorithms, designed to work with real-time control systems, handle these problems with a streamlined memory and computational efficient reorganization of the basic operations of the Kalman and Information Filters (KF & IF). The two versions designed to deal only with valid measurements are optimal solutions of the OOS problem, while the other two remaining are suboptimal algorithms able to handle corrupted data.

1. Introduction

The state of a complex control system is estimated by its fusion center with the data provided by its sensors. The time and order of arrival of the information at the fusion center depends on many factors, such as the physical distribution of the sensors and the communication network used to send the data. A difficult scenario occurs when the delays and the sequence of arrival of the information are not fixed, constituting the named Out-Of-Sequence Problem (OOSP) (Bar-Shalom, 2002). Another important problem happens when some data are provided by malfunctioning sensors or corrupted during the communication, and these behaviors are not modeled in the estimation algorithms. The fusion system is then in charge of assessing the validity of the information and deciding how to treat the erroneous data (Hall, 1992). Finally, both problems are usually aggravated in networked real-time control systems because the solution adopted to tackle them can affect the stability of its feedback loops (Hespanha, Naghhtabrizi, & Xu, 2007; Sinopoli et al., 2004).

In the case of sequential fusion algorithms, such as the KF and IF (Mutambara, 1998), there are three naïve solutions to deal with the OOSP. The first, rejecting the delayed information, is only appropriated with spurious delayed measurements because it increases the uncertainty and reduces the reliability of the control system (Hespanha et al., 2007; Sinopoli et al., 2004). The second, buffering all the data related with an instant before estimating its state (Lopez-Orozco, de la Cruz, Besada, & Rupiezed, 2000), is not valid for control systems where the response is needed before all the data are available. Finally, the third consists of storing the estimates of the state, the control signals, and the sensor data for all the time instances; rolling back to the time-stamp associated with the measurement which has just arrived, and re-starting the fusion process from that measurement (Kosaka, Meng, & Kak, 1993). This last solution lets the fusion center obtain the same results as if it had received the data without delays. However, it increases the memory needs of the fusion center and its computational overload introduces delays that can affect the controller. The new OOS versions of some estimators, such as Anxi, Diannong, Weidong, and Zhen (2005), Bar-Shalom (2002), Bar-Shalom, Mallick, Chen, and Washburn (2004), Challa, Evans, Wang, and Leggy (2002), Feng, Ge, and Wen (2008), Hilton, Martin, and Blair (1993), Ito, Tsujimichi, and Kosuge (1998), Lanzkron and Bar-Shalom (2004), Lu, Zhang, Wang, and Teo (2005), Mallick, Coraluppi, and Carthel (2001), Matveev and Savkin (2003), Nettleton and Durrant-Whyte

☆ This work is supported by the Community of Madrid under Project CAM-S-0505/DPI-0391 and by the Spanish Ministry of Science and Technology under Project DPI2009-14552-C02-01 and Project TECT2008-06732-C02-101. The work of E. Besada-Portas was also supported by the Spanish Postdoctoral Grant EX-2007-0915 associated with the Prince of Asturias Endowed Chair of the University of New Mexico, USA. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Giuseppe De Nicolao under the direction of Editor Ian R. Petersen.

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0005-1098/$ – see front matter
doi:10.1016/j.automatica.2011.02.030

(2001), Rheaume and Benaskeur (2008), Shen, Zhu, Song, and Luo (2009), WenHui, Lin, GuoHai, and AnXi (2006), Zhang, Li, and Chen (2002), Zhang, Li, and Chen (2003), Zhang, Xie, Zhang, and Soh (2004) and Zhang, Li, and Zhu (2005) for the KF, reduce these delays and memory needs.

The erroneous data problem can be tackled by including a validation step for testing if the measurements are coherent with the state of the system and rejecting them when they are not (Hall, 1992). When the OOSP is also present, the two problems interact. On the one hand, a validation test dependent on the estimate of the state that only considers the measurements available so far is influenced by their order of arrival. On the other hand, the estimates will depend on the data which have not been rejected because they successfully pass the validation test.

This paper presents a set of simple memory and computational efficient algorithms designed to estimate the state of MIMO linear control systems with additive non-correlated Gaussian noise in the transition and measurement models with all the valid available data received up to this point with random delays. They extend the range of applicability of our first OOS algorithm, named IFAsyn (IF for Asynchronous data) in Besada-Portas, Lopez-Orozco, and de la Cruz (2007) and Besada-Portas, Lopez-Orozco, Besada, and de la Cruz (2009) and hereafter IFAsyn-I (IFAsyn-version I), to systems that work without prior knowledge of the measurement time-stamps, assimilate multiple measurements with different time-stamps in a single update step, and incorporate a validation step to detect corrupted data. When the validation step is disabled, they find the same optimal solution as the KF. When the validation step is enabled, their results and the KF ones can differ, although our experiments show that the differences are negligible. The new algorithms, hereafter IFAsyn-(II, III, IV, V), also reduce the memory and computational needs of IFAsyn-I.

This paper also includes a comprehensive comparison of our OOS algorithms with many others. In short, all versions of IFAsyn are computationally efficient, simple to implement, and general in scope because they already: (1) include the control signal and (2) consider the multisensor case. Besides, IFAsyn-(II, III, IV, V) work without prior knowledge of the data time-stamps, IFAsyn-(III, V) assimilate multiple measurements in a single iteration, and IFAsyn-(IV, V) include a validation step.

The paper is organized as follows: Section 2 introduces some background, Section 3 describes our algorithms, Section 4 compares them with other algorithms, Section 5 analyzes the influence of the validation step in the results, and finally, Section 6 presents the conclusions.

2. Background

2.1. Problem statement

A discrete MIMO linear control system with additive Gaussian noise and S sensors is modeled[1] by Eq. (1), where $\boldsymbol{x}_t$ is the state of the system at time t; $\boldsymbol{z}_{s,t}$ the measurement of sensor s at time t; $\boldsymbol{u}_{t,t_P}$ the control signal applied from the previous time step t_P to the current t; $\boldsymbol{F}_{t,t_P}$ and $\boldsymbol{H}_{s,t}$ the transition and measurement matrices, and $\boldsymbol{\upsilon}_{t,t_P}$ and $\boldsymbol{\nu}_{s,t}$ random Gaussian variables with zero mean and covariances $\boldsymbol{Q}_{t,t_P}$ and $\boldsymbol{R}_{s,t}$.

$$\begin{aligned} \boldsymbol{x}_t &= \boldsymbol{F}_{t,t_P}\boldsymbol{x}_{t_P} + \boldsymbol{u}_{t,t_P} + \boldsymbol{\upsilon}_{t,t_P} \\ \boldsymbol{z}_{s,t} &= \boldsymbol{H}_{s,t}\boldsymbol{x}_t + \boldsymbol{\nu}_{s,t} \quad \text{with } s = 1:S. \end{aligned} \tag{1}$$

The objective of the fusion algorithm is to estimate the current system state and covariance ($\hat{\boldsymbol{x}}_{t|t}$, $\boldsymbol{P}_{t|t}$) given its original values ($\boldsymbol{x}_{0|0}$, $\boldsymbol{P}_{0|0}$), the model parameters and control signals $\{\boldsymbol{F}_{k,k_P}, \boldsymbol{Q}_{k,k_P}, \boldsymbol{H}_{s,k}, \boldsymbol{R}_{s,k}, \boldsymbol{u}_{k,k_P}\}$, and the data $\{\boldsymbol{\xi}_{s,k,a} = \boldsymbol{z}_{s,k} | a \geq k, a \leq t\}$ measured by sensor s at time k, which have arrived at the fusion center at time a ($a \geq k$), and which is already available ($a \leq t$). In addition, the algorithm is also responsible for detecting and rejecting erroneous data produced by failures not modeled in the sensor covariance matrices.

2.2. Estimating the state with non-delayed data ($\boldsymbol{\xi}_{s,t,t}$)

When the measurements are available without delays ($\boldsymbol{\xi}_{s,t,t}$), ($\hat{\boldsymbol{x}}_{t|t}$, $\boldsymbol{P}_{t|t}$) can be obtained by sequentially using the prediction and update steps of the KF (Mutambara, 1998). An equivalent approach, with the same two steps, is the IF (Mutambara, 1998). They operate in two different spaces, KF in the state space ($\hat{\boldsymbol{x}}_{t|t}$, $\boldsymbol{P}_{t|t}$) and IF in the information space ($\hat{\boldsymbol{y}}_{t|t}$, $\boldsymbol{Y}_{t|t}$), that are related by the state projection operation $\{\hat{\boldsymbol{y}}_{j|l} = \boldsymbol{P}_{j|l}^{-1}\hat{\boldsymbol{x}}_{j|l}, \boldsymbol{Y}_{j|l} = \boldsymbol{P}_{j|l}^{-1}\}(\perp_S)$. In each iteration they only need their previous time t_P space variables ($\hat{\boldsymbol{x}}_{t_P|t_P}$, $\boldsymbol{P}_{t_P|t_P}$ or $\hat{\boldsymbol{y}}_{t_P|t_P}$, $\boldsymbol{Y}_{t_P|t_P}$) and the current time t parameters and data ($\boldsymbol{F}_{t,t_P}, \boldsymbol{Q}_{t,t_P}, \boldsymbol{H}_{s,t}, \boldsymbol{R}_{s,t}, \boldsymbol{u}_{t,t_P}, \boldsymbol{\xi}_{s,t,t}$). Further, as the prediction (Eq. (2)) is simpler in the KF and the update of multiple measurements (Eq. (3)) is easier in the IF (Mutambara, 1998), the estimation problem can be solved by combining KF predictions, IF updates and state projections. Finally, the IF update can be divided in a projection of the measurement into the information space (3)($\perp_M$), the accumulation of all the projected measurements (3)(+), and the assimilation of the accumulated data with the previous information (3)(A).

$$\left.\begin{aligned} \hat{\boldsymbol{x}}_{t|t_P} &= \boldsymbol{F}_{t,t_P}\hat{\boldsymbol{x}}_{t_P|t_P} + \boldsymbol{u}_{t,t_P} \\ \boldsymbol{P}_{t|t_P} &= \boldsymbol{F}_{t,t_P}\boldsymbol{P}_{t_P|t_P}\boldsymbol{F}_{t,t_P}^T + \boldsymbol{Q}_{t,t_P} \end{aligned}\right\} (P) \tag{2}$$

$$\left.\begin{aligned} &\left\{\boldsymbol{i}_{s,t} = \boldsymbol{H}_{s,t}^T\boldsymbol{R}_{s,t}^{-1}\boldsymbol{\xi}_{s,t,t}, \boldsymbol{I}_{s,t} = \boldsymbol{H}_{s,t}^T\boldsymbol{R}_{s,t}^{-1}\boldsymbol{H}_{s,t}\right\} (\perp_M) \\ &\left\{\boldsymbol{i}_t = \sum_{s=1}^{S}\boldsymbol{i}_{s,t}, \boldsymbol{I}_t = \sum_{s=1}^{S}\boldsymbol{I}_{s,t}\right\} (+) \\ &\left\{\hat{\boldsymbol{y}}_{t|t} = \hat{\boldsymbol{y}}_{t|t_P} + \boldsymbol{i}_t, Y_{t|t} = \boldsymbol{Y}_{t|t_P} + \boldsymbol{I}_t\right\} (A) \end{aligned}\right\}. \tag{3}$$

To deal with erroneous measurements, a validation test that checks if $\boldsymbol{\xi}_{s,k,a}$ is coherent with the estimate of the state is sometimes included before the KF/IF update step (Hall, 1992). If the test is passed, $\boldsymbol{\xi}_{s,k,a}$ is used to update the estimate, otherwise it is rejected. Data association distances (Fukunaga, 1990) are used as validation tests because they are quick geometric methods to quantify the disagreement that exists between $\boldsymbol{\xi}_{s,k,a}$ and its predicted value ($\boldsymbol{H}_{s,k}\hat{\boldsymbol{x}}_{k|k_P}$). A typical validation test consists in comparing the obtained distance with a threshold l_s. The Mahalanobis distance $d_{s,k,a}$ in Eq. (4) is often used because it weighs the discrepancy between $\boldsymbol{\xi}_{s,k,a}$ and $\boldsymbol{H}_{s,k}\hat{\boldsymbol{x}}_{k|k_P}$ with the inverse of the covariance of the predicted measurement value ($\boldsymbol{H}_{s,k}\boldsymbol{P}_{k|k_P}\boldsymbol{H}_{s,k}^T + \boldsymbol{R}_{s,k}$). Thus, it grows with the discrepancy and decreases with the uncertainty. Further, $d_{s,k,a}$ follows a chi-square distribution $\chi_{n_s}^2$ of as many degrees of freedom n_s as the number of elements in $\boldsymbol{\xi}_{s,k,a}$. Consequently, the validation test only rejects valid measurements with a probability lower than α when the cumulative probability $P(\chi_{n_s}^2 < l_s) = 1 - \alpha/2$. See Johnson and Wichern (1998), for further details.

$$\begin{aligned} &d_{s,k,a} \leq l_s \\ &d_{s,k,a} = \boldsymbol{e}_{s,k,a}^T(\boldsymbol{H}_{s,k}\boldsymbol{P}_{k|k_P}\boldsymbol{H}_{s,k}^T + \boldsymbol{R}_{s,k})^{-1}\boldsymbol{e}_{s,k,a} \\ &\boldsymbol{e}_{s,k,a} = (\boldsymbol{\xi}_{s,k,a} - \boldsymbol{H}_{s,k}\hat{\boldsymbol{x}}_{k|k_P}). \end{aligned} \tag{4}$$

[1] Note that when t_P is substituted by $t-1$, the first expression of Eq. (1) becomes $\boldsymbol{x}_t = \boldsymbol{F}_{t,t-1}\boldsymbol{x}_{t-1} + \boldsymbol{u}_{t,t-1} + \boldsymbol{\upsilon}_{t,t-1}$, which is the usual equation in discrete linear systems, used to present IFAsyn-I in Besada-Portas et al. (2009, 2007). Our new notation better suits the aperiodicity of the events supported by the versions of IFAsyn introduced in this paper. The evaluation of $\boldsymbol{F}_{t,t_P}$, $\boldsymbol{u}_{t,t_P}$, $\boldsymbol{Q}_{t,t_P}$ depends on the system. For instance, when it is a discretized version of a continuous system modeled by $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{u}(t) + \boldsymbol{\upsilon}(t)$, the three variables can be calculated as $\boldsymbol{u}_{t,t_P} = \int_0^{t-t_P}\boldsymbol{\phi}(\tau)\boldsymbol{u}(\tau)\mathrm{d}\tau$, $\boldsymbol{F}_{t,t_P} = \phi(t-t_P)$ and $\boldsymbol{Q}_{t,t_P} = \int_0^{t-t_P}\boldsymbol{\phi}(\tau)\boldsymbol{Q}(\tau)\boldsymbol{\phi}^T(\tau)\mathrm{d}\tau$, where $\boldsymbol{x}(t)$ and $\dot{\boldsymbol{x}}(t)$ are the continuous state and its time derivate at time t, $\boldsymbol{A}$ the transition matrix, $\boldsymbol{u}(t)$ the control signal at t, $\boldsymbol{Q}(t)$ the covariance of the continuous zero mean noise $\boldsymbol{\upsilon}(t)$, and $\boldsymbol{\phi}(t) = \mathrm{e}^{\boldsymbol{A}t}$.