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Brief paper

A delay decomposition approach to $\mathcal{L}_2$–$\mathcal{L}_\infty$ filter design for stochastic systems with time-varying delay[☆]

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ABSTRACT

This paper investigates the problem of $\mathcal{L}_2$–$\mathcal{L}_\infty$ filter design for a class of stochastic systems with time-varying delay. The addressed problem is the design of a full order linear filter such that the error system is asymptotically mean-square stable and a prescribed $\mathcal{L}_2$–$\mathcal{L}_\infty$ performance is satisfied. In order to develop a less conservative filter design, a new Lyapunov-Krasovskii functional (LKF) is constructed by decomposing the delay interval into multiple equidistant subintervals, and a new integral inequality is established in the stochastic setting. Then, based on the LKF and integral inequality, the delay-dependent conditions for the existence of $\mathcal{L}_2$–$\mathcal{L}_\infty$ filters are obtained in terms of linear matrix inequalities (LMIs). The resulting filters can ensure that the error system is asymptotically mean-square stable and the peak value of the estimation error is bounded by a prescribed level for all possible bounded energy disturbances. Finally, two examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

The problem of state and signal estimation is central to a wide range of applications in signal processing and control. Over the past decades, considerable attention has been given to methods that are based on the minimization of the variance of the estimation error, i.e., the celebrated Kalman filtering approach (Anderson & Moore, 1979). One of the underlying assumptions of these methods is that the exogenous disturbances impinging on the system under consideration are stochastic in nature, but have known statistical properties. In many cases, however, the statistical nature of the external disturbances is not easily known. To solve this difficulty, some alternative filtering approaches have been developed, such as $\mathcal{H}_\infty$ filtering (Emara-Shabaik, Mahmoud, & Shi, 2010; Green & Limebeer, 1995; Liu & Wang, 2009; Simon, 2006), $\mathcal{L}_2$–$\mathcal{L}_\infty$ filtering (Grigoriadis & Watson, 1997; Palhares & Peres, 2000), and $\mathcal{L}_1$ filtering (Nagpal, Abedor, & Poolla, 1994; Tseng, 2006).

In recent years, the stochastic filtering and control problems with system models expressed by Itô-type stochastic differential equations have received considerable attention (see, e.g. Gershon, Limebeer, Shaked, & Yaesh, 2001; Hinrichsen & Pritchard, 1998; Xu & Chen, 2002a; Zhang, Chen, & Tseng, 2005, and the references therein). Such models are encountered in many areas of application, e.g., population models, nuclear fission and heat transfer, immunology, etc. (Mohler & Kolodziej, 1980). Meanwhile, time delay is often encountered in various engineering systems. In many cases, time delay is a source of instability and performance deterioration. The presence of time delay greatly complicates the stochastic filtering and control designs, and makes them more difficult (Deng, Shi, Yang, & Xia, 2010; Gu, 2001; Gu, Kharitonov, & Chen, 2003; Hale & Lunel, 1993; Han, 2005; Liu, Hu, & Tian, 2010; Wu, He, She, & Liu, 2004). Therefore, studying the filtering and control problems of stochastic systems with time delay is of theoretical and practical importance, and has attracted a rapid growing interest in the past few decades (Gao, Lames, & Wang, 2006; Liu, Wang, & Liu, 2007, 2008; Mao, 1996; Mao, Koroleva, & Rodkina, 1998; Xia, Xu, & Song, 2007; Xu & Chen, 2002b, 2003). In particular, the delay-independent $\mathcal{L}_2$–$\mathcal{L}_\infty$ filtering results for uncertain stochastic systems with time-varying delay were presented in Gao et al. (2006). The delay-dependent $\mathcal{L}_2$–$\mathcal{L}_\infty$ filtering results for stochastic systems with a constant time delay were given in Xia et al. (2007). Recently, the

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delay-range-dependent $\mathcal{L}_2$–$\mathcal{L}_\infty$ filtering design was also developed in Zhou, Chen, Li, and Lin (2009) for stochastic systems with time-varying interval delay. Despite these efforts, there is room for further improvement. Yet, how to further reduce the conservatism and computational load remains an important and challenging problem.

More recently, inspired by the discretized Lyapunov functional method proposed by Gu (2001), the delay decomposition approach has been developed for stability analyses of linear retarded and neutral systems (Han, 2009), linear systems with time-varying delays (Zhang & Han, 2009) and delayed T–S fuzzy systems (Zhao, Gao, Lames, & Du, 2009), respectively. It has been shown that this method can lead to less conservative results. Motivated by this fact, the delay decomposition approach will be employed to deal with the $\mathcal{L}_2$–$\mathcal{L}_\infty$ filtering design for a class of stochastic systems with time-varying delay in this work. First, a new LKF is constructed via delay decomposition, and a new integral inequality is established in the stochastic setting. Then, using the LKF and integral inequality, the delay-dependent conditions for the existence of $\mathcal{L}_2$–$\mathcal{L}_\infty$ filters are obtained in terms of linear matrix inequalities (LMIs), which can be efficiently solved using the existing LMI optimization techniques (Boyd, Ghaoui, Feron, & Balakrishnan, 1994; Gahinet, Nemirovski, Laub, & Chilali, 1995). The resulting filters can ensure that the error system is asymptotically mean-square stable and the peak value of the estimation error is bounded by a prescribed level for all possible bounded energy disturbances. Finally, two examples are given to show the effectiveness of the proposed method.

Notations: $\Re^n$ and $\Re^{m\times n}$ denote the n-dimensional Euclidean space and the set of real $m \times n$ matrix, respectively. For a real symmetric matrix X, $X > 0$ ($X \geq 0$) means that X is positive definite (positive semi-definite). The superscript "T" denotes the transpose of a matrix or a vector. The symbol "$*$" in a matrix stands for the transposed elements in the symmetric positions. $\lambda_{\min}(\cdot)$ means the minimal eigenvalue of a matrix. $\mathcal{E}\{\cdot\}$ denotes the expectation operator. $\mathcal{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. $|\cdot|$ refers to the Euclidean norm, and $\|\cdot\|$ stands for the usual $\mathcal{L}_2[0,\infty)$ norm.

2. Problem formulation

Consider a class of stochastic time-delay systems described by Itô-type stochastic retarded functional differential equations

$$\begin{aligned}\mathrm{d}x(t) &= [Ax(t) + A_d x(t-d(t)) + B\upsilon(t)]\mathrm{d}t \\ &\quad + [Mx(t) + M_d x(t-d(t))]\mathrm{d}w(t) \end{aligned} \tag{1}$$

$$\begin{aligned}\mathrm{d}y(t) &= [Cx(t) + C_d x(t-d(t)) + D\upsilon(t)]\mathrm{d}t \\ &\quad + [Nx(t) + N_d x(t-d(t))]\mathrm{d}w(t) \end{aligned} \tag{2}$$

$$z(t) = Hx(t) \tag{3}$$

$$x(t) = \varphi(t), \quad \forall t \in [-\tau, 0] \tag{4}$$

where $x(t) \in \Re^n$ is the state, $\upsilon(t) \in \Re^m$ is the disturbance input which belongs to $\mathcal{L}_2[0,\infty)$, $y(t) \in \Re^p$ is the measured output, $z(t) \in \Re^q$ is the signal to be estimated, and $w(t)$ is a one-dimensional Brownian motion satisfying $\mathcal{E}\{\mathrm{d}w(t)\} = 0$ and $\mathcal{E}\{\mathrm{d}w^2(t)\} = \mathrm{d}t$. $A, A_d, B, M, M_d, C, C_d, D, N, N_d$, and H are known constant matrices with appropriate dimensions. The state delay $d(t)$ is a time-varying differentiable function satisfying $0 \leq d(t) \leq \tau$ and $\left|\dot{d}(t)\right| \leq \mu < 1$, where $\tau > 0$ and $\mu \geq 0$ are constants. The initial condition $\phi(\cdot)$ is a vector-valued initial continuous function defined on the interval $[-\tau, 0]$.

Definition 1. The systems (1)–(4) with $\upsilon(t) = 0$ are said to be *mean-square stable* if for any $\varepsilon > 0$, there exists a $\delta > 0$ such that $\mathcal{E}\{|x(t)|^2\} < \varepsilon, t > 0$ when $\sup_{t\in[-\tau,0]} \mathcal{E}\{|\phi(t)|^2\} < \delta$. Moreover, if $\lim_{t\to\infty} \mathcal{E}\{|x(t)|^2\} = 0$, then the system is said to be *asymptotically mean-square stable*.

Assumption 1. The systems (1)–(4) with $\upsilon(t) = 0$ are asymptotically mean-square stable.

Suppose the following full order linear filter is proposed to estimate the signal $z(t)$:

$$\mathrm{d}\hat{x}(t) = A_F\hat{x}(t)\mathrm{d}t + B_F\mathrm{d}y(t), \quad \hat{x}(0) = 0 \tag{5}$$

$$\hat{z}(t) = C_F\hat{x}(t) \tag{6}$$

where $\hat{x}(t) \in \Re^n$ is the filter state, A_F, B_F and C_F are appropriately dimensioned filter matrices to be designed.

Define the estimation error by $e(t) = z(t) - \hat{z}(t)$. Then from (1)–(6), the following state-space equation for the estimation error is obtained:

$$\begin{aligned}\mathrm{d}\tilde{x}(t) &= [\bar{A}\tilde{x}(t) + \bar{A}_d K\tilde{x}(t-d(t)) + \bar{B}\upsilon(t)]\mathrm{d}t \\ &\quad + [\bar{M}\tilde{x}(t) + \bar{M}_d K\tilde{x}(t-d(t))]\mathrm{d}w(t) \end{aligned} \tag{7}$$

$$e(t) = \bar{H}\tilde{x}(t) \tag{8}$$

$$\tilde{x}(t) = \tilde{\phi}(t), \quad \forall t \in [-\tau, 0] \tag{9}$$

where $\tilde{x}(t) = [x^T(t) \quad \hat{x}^T(t)]^T$, $\tilde{\phi}(t) = [\phi^T(t) \quad 0]^T$, and

$$\bar{A} = \begin{bmatrix} A & 0 \\ B_FC & A_F \end{bmatrix}, \quad \bar{A}_d = \begin{bmatrix} A_d \\ B_FC_d \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ B_FD \end{bmatrix},$$

$$\bar{M} = \begin{bmatrix} M & 0 \\ B_FN & 0 \end{bmatrix}, \quad \bar{M}_d = \begin{bmatrix} M_d \\ B_FN_d \end{bmatrix}, \quad \bar{H} = \begin{bmatrix} H & -C_F \end{bmatrix},$$

$$K = \begin{bmatrix} I & 0 \end{bmatrix}.$$

For convenience, we introduce the following definition:

Definition 2. Given a scalar $\gamma > 0$, the error systems in (7)–(9) are said to be *asymptotically mean-square stable with the $\mathcal{L}_2$–$\mathcal{L}_\infty$ attenuation level γ* if the error systems in (7)–(9) with $\upsilon(t) = 0$ are asymptotically mean-square stable, and the estimation error under zero initial condition (i.e., $\phi(t) = 0$, $t \in [-\tau, 0]$) satisfies $\|e\|_{\mathcal{E}\infty} < \gamma \|\upsilon\|$ for all nonzero $\upsilon(t) \in \mathcal{L}_2[0,\infty)$, where $\|e\|_{\mathcal{E}\infty} \triangleq \sup_t \sqrt{\mathcal{E}\{|e(t)|^2\}}$.

The problem under consideration in this paper is to design a filter of the form (5)–(6) for the systems (1)–(4) such that the error systems in (7)–(9) are asymptotically mean-square stable with a prescribed $\mathcal{L}_2$–$\mathcal{L}_\infty$ attenuation level γ. Such a filter is regarded as a stochastic $\mathcal{L}_2$–$\mathcal{L}_\infty$ filter or an energy-to-peak filter.

The following lemma will be useful in the sequel.

Lemma 1. *Let n-dimensional vector functions $x(t)$, $\varphi(t)$, and $g(t)$ satisfy the stochastic differential equation*

$$\mathrm{d}x(t) = \varphi(t)\mathrm{d}t + g(t)\mathrm{d}w(t) \tag{10}$$

where $w(t)$ is a one-dimensional Brownian motion. For any constant matrix $Z \geq 0 \in \Re^{n\times n}$ and scalar $h > 0$, if the following integration is well defined, then

$$\begin{aligned}-h\int_{t-h}^{t} \varphi^T(s)Z\varphi(s)\mathrm{d}s &\leq \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -Z & Z \\ Z & -Z \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} \\ &\quad + 2\begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -Z \\ Z \end{bmatrix} \int_{t-h}^{t} g(s)\mathrm{d}w(s). \end{aligned} \tag{11}$$

Proof. It follows from (10) that

$$\int_{t-h}^{t} \varphi(s)\mathrm{d}s = x(t) - x(t-h) - \int_{t-h}^{t} g(s)\mathrm{d}w(s). \tag{12}$$