Available online at www.sciencedirect.com

ScienceDirect

Automatica 44 (2008) 1666–1675

automatica

www.elsevier.com/locate/automatica

Robustness with dynamic IQCs: An exact state-space characterization of nominal stability with applications to robust estimation[☆]

Carsten W. Scherer[a,*], I. Emre Köse[b]

[a] Delft Center for Systems and Control (DCSC), Delft University of Technology, Delft, The Netherlands
[b] Department of Mechanical Engineering, Boğazici University, Istanbul, Turkey

Received 24 March 2006; received in revised form 6 April 2007; accepted 16 October 2007
Available online 5 March 2008

Abstract

For robustness analysis with integral quadratic constraints, we formulate a new positivity condition on the solution of the corresponding linear matrix inequality which is necessary and sufficient for nominal stability of the underlying system. The application of this technical result is illustrated by a complete solution of the $\mathcal{L}_2$-gain and robust $\mathcal{H}_2$-estimator design problems if the uncertainties are characterized by dynamic integral quadratic constraints.

Keywords: IQC; Dynamic multipliers; Guaranteed stability; LMI-based synthesis; Robust estimation

1. Introduction

Integral quadratic constraints (IQCs) provide a general framework for stability analysis of feedback interconnections of LTI plants and uncertainty blocks as shown in Fig. 1. Many well-known results, such as robust stability analysis against structured time-varying parametric or dynamic non-linear uncertainties, can be viewed as special cases of the IQC approach. Although IQCs first appeared in different guises in the 1960s and 70s, mainly in the works of Yakubovich, a unifying framework was introduced in Megretski and Rantzer (1997). Since then research on IQCs has mostly focused on finding suitable multipliers for certain types of perturbation blocks such as an uncertain time-delay, multiple non-linearities, or rate-bounded parameters (D'Amato, Rotea, Megretski, & Jönsson, 2001; Jun & Safonov, 2002; Megretski & Rantzer, 1997).

If the nominal system is stable, verification of robust stability by IQCs can be cast, through the use of the Kalman–Yakubovich–Popov (KYP) lemma, as optimization problems which involve linear matrix inequality (LMI) constraints (Rantzer, 1996). Therefore, the IQC framework leads to computationally tractable conditions for checking robust stability.

However, the literature on employing the IQC framework for robust synthesis is scarce. One of the involved delicacies is related to guaranteeing nominal stability of the corresponding closed-loop system. Indeed, as a key ingredient in LMI-based synthesis approaches (such as $\mathcal{H}_\infty$- or $\mathcal{H}_2$-control), it is clearly understood how to enforce closed-loop stability by just imposing a positivity condition on the solution of the LMIs which specify performance. This fails for dynamic IQC performance specifications (Balakrishan, 2002).

The main technical goal of this paper is to formulate a new positivity condition on the LMI reformulation of the IQC frequency-domain inequality (FDI) which is both necessary and sufficient for nominal stability of the underlying system. As another key contribution, we provide a state-space analog of the frequency-domain algorithm given in Goh (1996) for the construction of a so-called IQC-factorization, and a clear understanding of the relation of these factorizations to the LMIs as they result from the KYP lemma. Initial results indicate the relevance of these insights for obtaining a complete solution of the gain-scheduling synthesis problem

[☆] This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Yasumasa Fujisaki under the direction of Editor Roberto Tempo.

[*] Corresponding author. Tel.: +31 15 27 85899; fax: +31 15 27 86679.
E-mail addresses: c.w.scherer@tudelft.nl (C.W. Scherer), koseemre@boun.edu.tr (I.E. Köse).

0005-1098/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2007.10.023

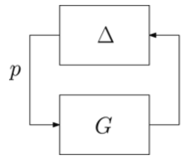

Fig. 1. Configuration for analysis.

with dynamic IQCs (Köse & Scherer, 2006). Despite its rather technical proof, the actual stability characterization is easy to formulate, and its relevance is demonstrated in this paper by providing solutions of the robust $\mathcal{L}_2$-gain and robust $\mathcal{H}_2$-estimation problems for uncertain LFT systems.

Several results on robust estimation using multipliers are available in the literature (de Souza & Trofino, 2000; Geromel, 1999; Geromel & de Oliveira, 2001; Li & Fu, 1997; Ravuri & Asada, 2000; Sun & Packard, 2005), but none of these allow the use of dynamic IQCs, which precludes to decrease conservatism or to handle interesting classes of uncertainties/non-linearities. Only the recent paper by Scorletti and Fromion (2006) addresses dynamic multipliers, but they are limited to the D/G-structure. Robust estimation for linear parameter-varying systems using parameter-dependent Lyapunov functions has also been investigated (Barbosa, de Souza, & Trofino, 2005; Geromel, de Oliveira, & Bernussou, 2002; Tuan, Apkarian, & Nguyen, 2003). However, apart from using particularly structured Lyapunov functions, these results invariably lead to parameter-dependent filters which require on-line measurement of the parameters for their implementation. Our stability characterization allows us to design estimators for LFT systems where the uncertainty is described by general dynamic multipliers and without the need for on-line parameter measurements.

The paper is structured as follows. Section 2 contains a brief recap of the IQC-theory and the corresponding LMIs. Section 3 comprises the two main technical results. In Section 4, we demonstrate the application of our stability characterization to robust estimation, both with the $\mathcal{L}_2$-gain and a generalization of the $\mathcal{H}_2$-norm as a performance measure. The numerical example in Section 5 demonstrates how the proposed solution reduces conservatism. Appendices A–E contain all the proofs.

2. Recap of IQC robust stability analysis

Suppose that G is a stable LTI transfer matrix, and that we are interested in verifying the stability of the interconnection of G with a bounded, causal uncertainty block Δ as in Fig. 1. With a matrix-valued function $\Pi = \Pi^*$ that is essentially bounded on the extended imaginary axis $\mathbb{C}^0$, recall that $\Delta : \mathcal{L}_2[0, \infty) \to \mathcal{L}_2[0, \infty)$ is said to satisfy the IQC defined by Π if

$$\int_{-\infty}^{\infty} (\star)^* \Pi(i\omega) \begin{pmatrix} \hat{v}(i\omega) \\ \widehat{\Delta v}(i\omega) \end{pmatrix} \mathrm{d}\omega \geq 0 \quad \forall v \in \mathcal{L}_2[0, \infty), \tag{1}$$

where $\widehat{\ }$ denotes the Fourier transform. Note that $(\star)$ is used for expressions that can be deduced by symmetry. Once the characteristics of Δ have been described through an IQC, the stability of the feedback interconnection of G and Δ in Fig. 1 can be verified as follows.

Theorem 1 (Megretski & Rantzer, 1997). *Suppose G is stable and*

(i) *the feedback interconnection of $\tau\Delta$ and G is well-posed for all $\tau \in [0, 1]$,*
(ii) *$\tau\Delta$ satisfies the IQC defined by Π for all $\tau \in [0, 1]$,*
(iii) *G satisfies*

$$\begin{pmatrix} G \\ I \end{pmatrix}^* \Pi \begin{pmatrix} G \\ I \end{pmatrix} \prec 0 \quad \text{on } \mathbb{C}^0. \tag{2}$$

Then, the feedback interconnection of G and Δ is stable.

From now on we restrict ourselves to practically important real-rational multipliers Π that are bounded on $\mathbb{C}^0$. If all conditions in Theorem 1 are satisfied for Π, they hold as well for $\Pi + \epsilon I$ with some sufficiently small $\epsilon > 0$. Since (ii) in Theorem 1 implies $\Pi_{11} \succeq 0$ on $\mathbb{C}^0$, we can hence assume without loss of generality that

$$\Pi_{11} \succ 0 \quad \text{on } \mathbb{C}^0. \tag{3}$$

Since Π is bounded, one can easily construct a symmetric M and a proper and stable transfer matrix Ψ such that

$$\Pi = \Psi^* M \Psi. \tag{4}$$

Since Ψ is typically tall, we stress that such factorizations are highly non-unique. Now partition the columns of Ψ as $(\Psi_1 \ \ \Psi_2)$ compatibly with the rows of $\mathrm{col}(G, I)$. If $(A_\Psi, (B_{\Psi 1} \ B_{\Psi 2}), C_\Psi, D_\Psi)$ is a stable state-space realization of Ψ, we can determine a Kalman decomposition of $(A_\Psi, B_{\Psi 1})$ and continue with the realizations

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right], \qquad (\Psi_1 \ \ \Psi_2) = \left[\begin{array}{cc|cc} A_1 & A_3 & B_1 & B_3 \\ 0 & A_2 & 0 & B_2 \\ \hline C_1 & C_2 & D_1 & D_2 \end{array}\right] \tag{5}$$

where A_1, A_2 are stable and (A_1, B_1) is controllable. Then $\Psi_1 G + \Psi_2$ admits the state-space description

$$\left[\begin{array}{ccc|c} A_1 & A_3 & B_1 C & B_1 D + B_3 \\ 0 & A_2 & 0 & B_2 \\ 0 & 0 & A & B \\ \hline C_1 & C_2 & D_1 C & D_1 D + D_2 \end{array}\right] =: \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right]. \tag{6}$$

Moreover, if A has no eigenvalues in $\mathbb{C}^0$, the FDI (2) is equivalent to the existence of some $X = X^{\mathrm{T}}$ for which

$$\begin{pmatrix} I & 0 \\ \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}^{\mathrm{T}} \underbrace{\begin{pmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & M \end{pmatrix}}_{=:\mathcal{M}(X,M)} \begin{pmatrix} I & 0 \\ \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix} \prec 0. \tag{7}$$

Since M is indefinite, stability of $\mathcal{A}$ neither implies, nor is implied by positive definiteness of X in general (Balakrishan, 2002). However, characterizing stability of $\mathcal{A}$ or (equivalently) of A by some suitable condition on X is a key ingredient for all LMI-based controller synthesis techniques (see e.g. Scherer, Gahinet, and Chilali (1997) and references therein) in order to guarantee closed-loop stability.