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Brief paper

Fault tolerant control using sliding modes with on-line control allocation☆

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Abstract

This paper proposes an on-line sliding mode control allocation scheme for fault tolerant control. The effectiveness level of the actuators is used by the control allocation scheme to redistribute the control signals to the remaining actuators when a fault or failure occurs. The paper provides an analysis of the sliding mode control allocation scheme and determines the nonlinear gain required to maintain sliding. The on-line sliding mode control allocation scheme shows that faults and even certain total actuator failures can be handled directly without reconfiguring the controller. The simulation results show good performance when tested on different fault and failure scenarios.

Keywords: Fault tolerant control; Control allocation; Reconfigurable control; Sliding modes

1. Introduction

In most safety critical systems e.g. passenger aircraft (Brière & Traverse, 1993) and modern fighter aircraft (Forssell & Nilsson, 2005), there is actuator redundancy. This allows freedom to design fault tolerant control (FTC) systems to maintain stability and acceptable performance during faults and failures. Control allocation (CA) is one approach to manage the actuator redundancy for different control strategies handling actuator faults (see for example Buffington, Chandler, and Pachter (1999), Davidson, Lallman, and Bundick (2001)). There is extensive literature on CA which discusses different algorithms, approaches and applications: Enns (1998) discusses two (broadly) linked approaches (linear and quadratic programming) for CA based on finding the 'best solution' to a system of linear equations. The work in Härkegård and Glad (2005) compares control allocation with optimal control design for distributing the control effort among redundant actuators. In Buffington and Enns (1996) the authors demonstrate that feedback control systems with redundant actuators can be reduced to a feedback control system without redundancy using a special case of CA known as 'daisy chaining'. In this approach, a subset of the actuators, regarded as the primary actuators are used first, then secondary actuators are used if the primary actuators reach saturation. Other CA approaches taking into account actuator limits are discussed in Bordignon and Durham (1995) and Boskovic and Mehra (2002).

The work in Buffington et al. (1999) and Davidson et al. (2001) uses CA as a means for fault tolerant control (FTC). The benefits of CA is that the controller structure does not have to be reconfigured in the case of faults and it can deal directly with total actuator failures without requiring reconfiguration/accommodation of the controller: the CA scheme automatically redistributes the control signal. This is the facet of CA that will be explored in this paper. The work in Shtessel, Buffington, and Banda (2002) and Wells and Hess (2003) provides practical examples of the combination of sliding mode control (SMC) and CA for FTC. The work by Shin et al. (Shin, Moon, & Kim, 2005) uses control allocation ideas, but formulates the problem from an adaptive controller point of view. However neither of these papers provide a detailed stability analysis and discuss sliding mode controller design issues when using control allocation. Recent work by Corradini, Orlando, and Parlangeli (2005) shows that total failures can be dealt with by SMC schemes provided that there is enough redundancy in the system. However Corradini et al. (2005) considers exact duplication of actuators to achieve redundancy,

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whereas in many over actuated systems, the redundant actuators do not have identical dynamics to the 'primary' actuators.

In this paper, a combination of SMC and CA will be explored to achieve FTC. A rigorous design procedure is developed from a theoretical perspective. The control strategy uses the effectiveness level of the actuators, and redistributes the control to the remaining actuators when faults/failures occur. This is the novelty of this paper compared to the work in Corradini et al. (2005), Shtessel et al. (2002) and Wells and Hess (2003).

2. Controller design

2.1. Problem formulation

This paper considers a situation where a fault associated with the actuators develops in a system. It will be assumed that the system subject to actuator faults or failures, can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) - BK(t)u(t), \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. The effectiveness gain $K(t) = \operatorname{diag}(k_1(t), \ldots, k_m(t))$ where the $k_i(t)$ are scalars satisfying $0 \le k_i(t) \le 1$. These scalars model a decrease in effectiveness of a particular actuator. If $k_i(t) = 0$, the ith actuator is working perfectly whereas if $k_i(t) > 0$, a fault is present, and if $k_i(t) = 1$ the actuator has failed completely. In this paper, information about $K(t)$ will be incorporated into the allocation algorithm. In most CA strategies, the control signal is distributed equally among all the actuators (Shin et al., 2005) or distributed based on the limits (position and rate) of the actuators (Davidson et al., 2001).

In the literature, the assumption that $\operatorname{Rank}(B) = l < m$ is often employed so that B can be factorized into $B = B_\nu N$ where $N \in \mathbb{R}^{m\times l}$. For many systems the assumption, is not valid. However, the system states can always be reordered, and the matrix B from (1) can be partitioned as:

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \tag{2}$$

where $B_1 \in \mathbb{R}^{(n-l)\times m}$ and $B_2 \in \mathbb{R}^{l\times m}$ has rank l. The partition is in keeping with the notion of splitting the control law from the control allocation task (Davidson et al., 2001; Härkegård & Glad, 2005). Here it is assumed that the matrix B_2 represents the dominant contribution of the control action on the system, while B_1 generally will have elements of small magnitude compared with $\|B_2\|$. Compared to the work in Shin et al. (2005) where it is assumed that $B_1 = 0$, here $B_1 \neq 0$ will be considered explicitly in the controller design and in the stability analysis. It will be assumed without loss of generality that the states of the system in (1) have been transformed so that $B_2 B_2^{\mathrm{T}} = I_l$ and therefore $\|B_2\| = 1$. This is always possible since $\operatorname{rank}(B_2) = l$ by construction. As in Alwi and Edwards (2006), let the 'virtual control' $\nu(t)$ be defined as

$$\nu(t) := B_2 u(t) \tag{3}$$

so that

$$u(t) = B_2^{\dagger}\nu(t), \tag{4}$$

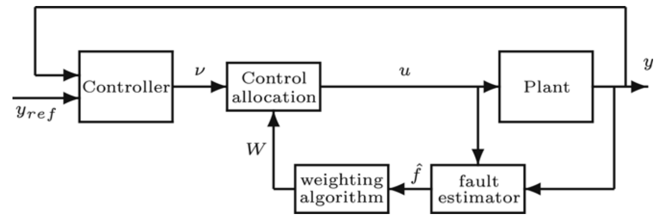

Fig. 1. Control allocation strategy.

where the pseudo inverse is chosen as

$$B_2^{\dagger} := W B_2^{\mathrm{T}} (B_2 W B_2^{\mathrm{T}})^{-1} \tag{5}$$

and $W \in \mathbb{R}^{m\times m}$ is a symmetric positive definite (s.p.d) diagonal weighting matrix. It can be shown the pseudo-inverse in (5) arises from the optimization problem

$$\min_{u(t)} u(t)^{\mathrm{T}} W^{-1} u(t) \quad \text{subject to } B_2 u(t) = \nu(t). \tag{6}$$

In this paper a novel choice of weighting matrix W will be considered. Specifically, W has been chosen as

$$W := I - K \tag{7}$$

and so $W = \operatorname{diag}\{w_1, \ldots, w_m\}$ where $w_i = 1 - k_i$. Note in a fault free situation $W = I$. As $k_i \to 1$, $w_i \to 0$ and so the associated component u_i in (6) is weighted heavily since $\frac{1}{w_i}$ becomes large.

Fig. 1 illustrates the FTC control allocation strategy. The control allocation will depend on the effectiveness of the actuators. The information necessary to compute W on-line can be supplied by a fault reconstruction scheme as described in Edwards, Spurgeon, and Patton (2000) and Tan and Edwards (2003) for example, or by using a measurement of the actual actuator deflection compared to the demand which is available in many systems e.g. passenger aircraft (Brière & Traverse, 1993). Alternatively other fault reconstruction schemes based on Kalman filters (Zhang & Jiang, 2002) can be used. From (7) if an actuator fault occurs, the weighting W will be changed on-line and the control input $u(t)$ is reallocated to minimize the use of the faulty control surface. In the event of total failure of the ith control surface, $k_i \to 1$ and therefore the ith component of W^{-1} becomes large. Hence, $u_i(t)$ is totally re-routed to the other actuators (provided there is enough redundancy in the system).

In this paper, sliding mode control (SMC) techniques (Edwards & Spurgeon, 1998), have been used to synthesize the 'virtual control' $\nu(t)$. Define a switching function $s(t) : \mathbb{R}^n \to \mathbb{R}^l$ to be

$$s(t) = Sx(t)$$

where $S \in \mathbb{R}^{l\times n}$ and $\det(SB_\nu) \neq 0$. Let $\mathcal{S}$ be the hyperplane defined by $\mathcal{S} = \{x(t) \in \mathbb{R}^n : Sx(t) = 0\}$. If a control law can be developed which forces the closed-loop trajectories onto the surface $\mathcal{S}$ in finite time and constrains the states to remain there, then an ideal sliding motion is said to have been attained (Edwards & Spurgeon, 1998). First define

$$\hat{\nu}(t) := (B_2 W^2 B_2^{\mathrm{T}})(B_2 W B_2^{\mathrm{T}})^{-1}\nu(t) \tag{8}$$