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H_∞ filtering for discrete-time systems with randomly varying sensor delays[☆]

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Abstract

This paper investigates an H_∞ filtering problem for discrete-time systems with randomly varying sensor delays. The stochastic variable involved is a Bernoulli distributed white sequence appearing in measured outputs. This measurement mode can be used to characterize the effect of communication delays and/or data-loss in information transmissions across limited bandwidth communication channels over a wide area. H_∞ filtering of this class of systems is used to design a filter using the measurements with random delays to ensure the mean-square stochastic stability of the filtering error system and to guarantee a prescribed H_∞ filtering performance. A sufficient condition for the existence of such a filter is presented in terms of the feasibility of a linear matrix inequality (LMI). Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

Keywords: Communication network; Data-loss; H_∞ filtering; Random delays; Stochastic stability

1. Introduction

Estimation of dynamic systems has found many practical applications and has attracted a lot of attention during the last decades. H_∞ filtering is introduced as an alternative to classical Kalman filtering when the statistical property of noise sources is unknown or unavailable (Nagpal & Khargonekar, 1991). H_∞ filtering is concerned with the design of estimators which ensure a bound on the $\mathcal{L}_2$-induced gain from disturbance signals to estimation errors. Over the past decades, various approaches, such as the interpolation approach, the Riccati equation-based approach, and the LMI-based approach have been developed to deal with the H_∞ filtering problem in various settings such as deterministic systems with uncertainties and/or delays as well as various stochastic systems (Xie, Liu, Zhang, & Zhang, 2004). Recently, the study of the H_∞ filtering problem for systems with delays has gained growing interest. A delay-independent H_∞ filtering result for discrete-time systems with multiple time delays has been given in Palhares, de Souza, and Peres (2001) while delay-dependent results for this problem have been reported more recently in Gao, Lam, Xie, and Wang (2004) and Gao and Wang (2005).

It is noted that the time delays are assumed to be deterministic in the literature mentioned above. However, they may occur in a randomly varying way in many practical applications as pointed out in Wang, Ho, and Liu (2004). Recently, there has been some attention to the research of systems with randomly varying delays. A randomly varying delayed sensor mode was first introduced in Ray (1994). Since then, some results for randomly delayed systems have been reported in Wang et al. (2004), Yaz and Ray (1996) and Yang, Wang, Hung, and Gani (2006). A variance-constrained filtering approach was proposed for systems with random sensor delays in Wang et al. (2004). And more recently, the authors in Yang et al. (2006) investigated the H_∞ control problem for this class of systems. In the meantime, the H_∞ filtering and control problems of stochastic systems have also attracted a lot of attention over the past few years (Hinrichsen & Pritchard, 1998). In Xu and Chen (2003), an LMI-based filter design approach was proposed for impulsive stochastic systems, and

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based on the projection lemma, the reduced-order H_∞ filtering problem for a class of stochastic systems was investigated in Xu and Chen (2002). The authors of Berman and Shaked (2006) presented an H_∞ control scheme for a class of discrete-time nonlinear stochastic systems more recently.

Motivated by the works in Wang et al. (2004) and Yang et al. (2006), this paper focuses on the H_∞ filtering problem for systems with randomly varying sensor delays. We are interested in designing filters such that for all randomly varying sensor delays, the filtering error system is exponentially mean-square stable and a prescribed H_∞ filtering performance is achieved.

This paper is organized as follows. Section 2 formulates the H_∞ filtering problem. In Section 3, a novel H_∞ filtering approach is proposed. A numerical example is given to demonstrate the effectiveness of the proposed method in Section 4, which is followed by conclusions in Section 5.

Notations: Throughout this paper, Z^+ denotes the set of positive integers; R^n denotes the n-dimensional Euclidean space; $R^{m\times n}$ denotes the set of all $m \times n$ real matrices. A real symmetric matrix $P > 0 (\geq 0)$ denotes P being a positive definite (or positive semi-definite) matrix, and $A > (\geq) B$ means $A - B > (\geq) 0$. I denotes an identity matrix of appropriate dimension. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations. The superscript 'τ' represents the transpose. For a matrix N, $N^{-\tau}$ stands for the transpose of matrix N^{-1}. A star '$*$' is used as an ellipsis for a corresponding transposed block matrix that is induced by symmetry. The notation $l_2[0,\infty)$ represents the space of square summable infinite vector sequences with the usual norm $\|\cdot\|_2$. Suppose that the vectors $v_k \in R^{\bar{n}}$, a sequence $v = \{v_k\} \in l_2[0,\infty)$ if $\|v\|_2 = \sqrt{\sum_{k=1}^{\infty} v_k^\tau v_k} < \infty$. Prob{.} stands for the occurrence probability of an event; $E\{.\}$ denotes the expectation operator with respect to some probability measure.

2. Problem formulation

Consider a discrete-time system Σ:

$$x_{k+1} = Ax_k + A_\omega \omega_k \tag{1}$$

$$z_k = Lx_k + L_\omega \omega_k \tag{2}$$

where $x_k \in R^n$ is the state; $\omega_k \in R^{\bar{p}}$ is the deterministic disturbance signal in $l_2[0,\infty)$; $z_k \in R^q$ is the signal to be estimated; A, A_ω, L and L_ω are known constant matrices with compatible dimensions. The measurement with random delays is given by

$$y_k = Cx_k \tag{3}$$

$$y_{ck} = (1-\theta_k) y_k + \theta_k y_{k-1} \tag{4}$$

where $y_k \in R^p$ is the output, $y_{ck} \in R^p$ is the measured output, C is a known matrix, and the stochastic variable θ_k is a Bernoulli distributed white sequence taking the values of 0 and 1 with

$$\text{Prob}\{\theta_k = 1\} = E\{\theta_k\} = \rho \tag{5}$$

$$\text{Prob}\{\theta_k = 0\} = E\{1-\theta_k\} = 1-\rho \tag{6}$$

where $\rho \in [0,1]$ and is a known constant.

Remark 1. The system measurements with varying sensor delay modeled in (3) and (4) was first introduced in Ray (1994) and has been used to characterize the effect of communication delays and/or data-loss in information transmissions across limited bandwidth communication channels over a wide area such as navigating a vehicle based on the estimations from a sensor web of its current position and velocity (Sinopoli et al., 2004). The output y_k produced at a time k is sent to the observer through a communication channel. If no packet-loss occurs, the measurement output y_{ck} takes value y_k; otherwise, the measurement output y_{ck} takes value y_{k-1}. When the probability of event packet-loss occurring is assumed as ρ, the measurement output y_{ck} in (4) thus takes the value y_k with probability $1-\rho$, and the value y_{k-1} with probability ρ.

For the delayed sensor mode (4), we assume that $x_{-1} = 0$, which implies from (3) that $y_{-1} = 0$.

We consider the following filter for the estimation of z_k:

$$\left.\begin{aligned} \hat{x}_{k+1} &= A_f \hat{x}_k + B_f y_{ck} \\ \hat{z}_k &= C_f \hat{x}_k + D_f y_{ck} \end{aligned}\right\} \tag{7}$$

where $\hat{x}_k \in R^n$ and $\hat{z}_k \in R^q$. A_f, B_f, C_f and D_f are matrices to be determined. Combining (1)–(4) and (7) the filtering error dynamics can be represented as $\tilde{\Sigma}$:

$$\left.\begin{aligned} \bar{x}_{k+1} &= \mathcal{A}(\theta_k)\bar{x}_k + \mathcal{A}_1(\theta_k) H \bar{x}_{k-1} + \mathcal{A}_\omega(\theta_k)\omega_k \\ \bar{z}_k &= \mathcal{L}(\theta_k)\bar{x}_k + \mathcal{L}_1(\theta_k) H \bar{x}_{k-1} + \mathcal{L}_\omega(\theta_k)\omega_k \end{aligned}\right\} \tag{8}$$

where

$$\bar{x}_k = \begin{bmatrix} x_k^\tau & \hat{x}_k^\tau \end{bmatrix}^\tau, \quad \bar{z}_k = z_k - \hat{z}_k, \quad H = \begin{bmatrix} I, & 0 \end{bmatrix} \tag{9}$$

$$\left.\begin{aligned} &\mathcal{A}(\theta_k) = \begin{bmatrix} A & 0 \\ (1-\theta_k)B_f C & A_f \end{bmatrix}, \quad \mathcal{A}_\omega = \begin{bmatrix} A_\omega \\ 0 \end{bmatrix} \\ &\mathcal{A}_1(\theta_k) = \begin{bmatrix} 0 \\ \theta_k B_f C \end{bmatrix}, \quad \mathcal{L}_1(\theta_k) = -\theta_k D_f C \\ &\mathcal{L}(\theta_k) = \begin{bmatrix} L - (1-\theta_k) D_f C, & -C_f \end{bmatrix}, \quad \mathcal{L}_\omega = L_\omega. \end{aligned}\right\} \tag{10}$$

It is noted that the filtering error dynamics $\tilde{\Sigma}$ is a system with stochastic parameters since some of the parametric matrices in (10) are associated with the stochastic variable θ_k. We adopt the notion of stochastic stability in the mean-square sense from Yang et al. (2006) to formulate our filtering problem.

Definition 1. The filtering error dynamics $\tilde{\Sigma}$ is said to be exponentially mean-square stable if with $\omega_k \equiv 0$, there exist constants $\alpha > 0$ and $\tau \in (0,1)$ such that

$$E\{\|\bar{x}_k\|^2\} \leq \alpha \tau^k E\{\|\bar{x}_0\|^2\}, \quad \text{for all } \bar{x}_0 \in R^{2n},\ k \in Z^+.$$

The H_∞ filtering problem addressed in this paper is to design a filter in the form of (7) such that for a given scalar γ and all nonzero ω_k, the filtering error system $\tilde{\Sigma}$ is exponentially mean-square stable and under the zero initial condition, the filtering error $\bar{z}_k$ satisfies

$$\sum_{k=0}^{\infty} E\{\|\bar{z}_k\|^2\} \leq \gamma^2 \sum_{k=0}^{\infty} \|\omega_k\|^2. \tag{11}$$

In such a case, the filtering error system is said to be exponentially mean-square stable with H_∞ filtering performance γ.