Automatica 47 (2011) 411–417

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Guaranteed cost control of stochastic uncertain systems with slope bounded nonlinearities via the use of dynamic multipliers[☆]

Ian R. Petersen [*]

School of Engineering and Information Technology, University of New South Wales at the Australian Defence Force Academy, Canberra, Australia

ARTICLE INFO

Article history:
Available online 17 December 2010

Keywords:
Nonlinear control
IQCs
Robust control
Uncertain systems
Dynamic multipliers

ABSTRACT

This paper presents a new approach to constructive output feedback robust nonlinear guaranteed cost controller design. The approach involves a class of controllers which include copies of the slope bounded nonlinearities occurring in the plant. Dynamic multipliers are introduced to exploit these repeated nonlinearities. The linear part of the controller is synthesized using minimax LQG control theory.

1. Introduction

This paper presents an approach to constructive output feedback robust nonlinear guaranteed cost controller design based on the use of Integral Quadratic Constraints (IQCs) and dynamic multipliers which exploit repeated slope bounded nonlinearities. These dynamic multipliers and IQCs are derived from the results of D'Amato, Rotea, Megreteski, and Jonsson (2001) and Kulkarni and Safonov (2002). The results are limited to nonlinear systems with known slope bounded nonlinearities of the type considered in Ref. Arcak and Kokotovic (2001). This means that the known nonlinearities must be globally Lipschitz. However, the unknown nonlinearities, which are treated as uncertainties satisfying IQCs can be more general; e.g., see Petersen, Ugrinovskii, and Savkin (2000).

The approach presented provides a systematic methodology for constructing robust nonlinear controllers for a class of uncertain nonlinear systems. The approach is based on the minimax LQG theory of Petersen et al. (2000) and Ugrinovskii and Petersen (2001). The results of the paper are related to a number of earlier papers in which the circle or Popov criteria have been used as tools in nonlinear feedback design; e.g., see Arcak, Larsen, and Kokotovic (2003) and the references therein. However in our case, we exploit the full power of the minimax LQG methodologies in order to be able to handle the issues of control system performance and measurement feedback in a systematic way.

The fundamental idea behind our approach is to modify the standard IQC approach to robust control by including a copy of the nonlinearity in the controller as shown in Fig. 1. This approach is similar to that used in the Linear Parameter Varying approach to controller design; e.g., see Packard (1994). Also, the idea of using a copy of the nonlinearity in nonlinear observer design was previously used in the paper (Arcak & Kokotovic, 2001). In our case, we combine both nonlinearities into the plant and then use IQCs and dynamic multipliers which exploit the repeated nonlinearity (in order to reduce conservatism); see Fig. 2. Our approach enables us to use minimax LQG control theory to construct the linear part of the controller and then the nonlinear controller is constructed by including a copy of the plant nonlinearity. The main result of this paper extends the ideas of previous papers by the author in this area (Petersen, 2008, 2009) to allow for dynamic multipliers which exploit the repeated nonlinearities and to consider the case of infinite horizon guaranteed cost controllers.

2. Problem statement

We consider a nonlinear stochastic uncertain system defined as follows. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space such that $\Omega = \mathbf{R}^n \times \mathbf{R}^l \times C([0,\infty), \mathbf{R}^f)$, the probability measure P is defined as the product of a given probability measure on $\mathbf{R}^n \times \mathbf{R}^l$ and the standard Wiener measure on $C([0,\infty), \mathbf{R}^f)$. Also, let $W(\cdot)$ be a f-dimensional standard Wiener process, $x_0 : \Omega \to \mathbf{R}^n$ be a Gaussian random variable with mean $\check{x}_0$ and non-singular covariance matrix Y_0. Also, the random variable x_0 and the Wiener process $W(\cdot)$ are assumed to be stochastically independent on $(\Omega, \mathcal{F}, P)$. On this probability space, we consider the system

[☆] This work was supported by the Australian Research Council. The material in this paper was partially presented at the 45th IEEE Conference on Decision and Control, December 13–15, 2006, San Diego, CA, USA. This paper was recommended for publication in revised form by Associate Editor Murat Arcak under the direction of Editor Andrew R. Teel.

[*] Tel.: +61 2 62688446; fax: +61 2 62688443.
E-mail addresses: i.r.petersen@gmail.com, i.petersen@adfa.edu.au.

0005-1098/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2010.11.002

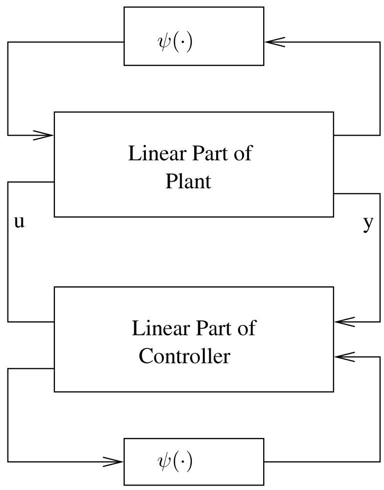

Fig. 1. Nonlinear system with nonlinear controller.

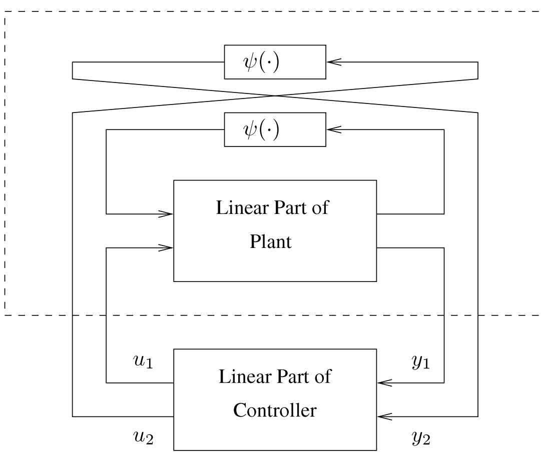

Fig. 2. Nonlinear system and linear controller redrawn with repeated nonlinearity.

driven by the noise input $W(\cdot)$ and a control input $u(\cdot)$, as described by the stochastic differential equations:

$$\begin{aligned} dx(t) &= Ax(t)dt + \left[\sum_{i=1}^{g} B_{1,i}\mu_i(t) + \check{B}_1\xi(t)\right]dt \\ &\quad + B_2u(t)dt + B_1dW(t); \qquad x(0) = x_0; \\ \zeta(t) &= \check{C}_1x(t) + \check{D}_{12}u(t); \\ \nu_i(t) &= C_{1,i}x(t) + D_{12,i}u(t); \quad i = 1, 2, \ldots, g; \\ dy(t) &= C_2x(t)dt + \left[\sum_{i=1}^{g} D_{21,i}\mu_i(t) + \check{D}_{21}\xi(t)\right]dt \\ &\quad + D_{21}dW(t); \qquad y(0) = 0 \end{aligned} \tag{1}$$

where $x \in \mathbf{R}^n$ is the *state*, $u \in \mathbf{R}^m$ is the *control input*, $\zeta \in \mathbf{R}^h$ is the *uncertainty output*, $\nu_1 \in \mathbf{R}, \ldots, \nu_g \in \mathbf{R}$ are the *nonlinearity outputs*, $\xi \in \mathbf{R}^r$ is the *uncertainty input*, $\mu_1 \in \mathbf{R}, \ldots, \mu_g \in \mathbf{R}$ are the *nonlinearity inputs*, and $y \in \mathbf{R}^l$ is the *measured output*. Also, the nonlinearity inputs are related to the nonlinearity outputs by the following nonlinear relations:

$$\mu_i(t) = \psi_i(\nu_i(t)) \quad \forall i = 1, 2, \ldots, g \tag{2}$$

which satisfy the following slope bound conditions:

$$\alpha_i < \frac{\psi_i(\nu_1) - \psi_i(\nu_2)}{\nu_1 - \nu_2} < \beta_i \tag{3}$$

for all ν_1, ν_2 with $\nu_1 \neq \nu_2$ and for all $i = 1, 2, \ldots, g$. Here the α_i and β_i are given constants.

The uncertainty in the system is block structured H^∞ norm bounded linear time-invariant uncertainty relating the uncertainty inputs and outputs as follows:

$$\xi(s) = \Delta(s)\zeta(s) \tag{4}$$

where $\Delta(s) = \mathrm{diag}\{\Delta_1(s), \ldots, \Delta_k(s)\}$ and

$$\|\Delta_i(s)\|_\infty \leq 1 \quad \forall i \in \{1, \ldots, k\}. \tag{5}$$

2.1. Cost functional and controller

Associated with the system (1), consider a quadratic cost functional of the form

$$J(u) = \limsup_{T\to\infty} \frac{1}{2T}\mathbf{E}\int_0^T (x'Rx + u'Gu)dt. \tag{6}$$

In (6), $R \in \mathbf{R}^{n\times n}$ and $G \in \mathbf{R}^{m\times m}$ are positive-definite symmetric matrices. Also, the class of controllers considered are nonlinear output feedback controllers:

$$\begin{aligned} dx_c(t) &= \left(A_cx_c(t) + \sum_{i=1}^{g}\tilde{G}_{ci}\tilde{\mu}_i(t)\right)dt + B_cdy(t); \\ x_c(0) &= x_{c0}; \\ \tilde{\nu}_i(t) &= \tilde{K}_{ci}x_c(t); \quad i = 1, 2, \ldots, g; \\ u(t) &= C_cx_c(t) \end{aligned} \tag{7}$$

where

$$\tilde{\mu}_i(t) = \psi_i(\tilde{\nu}_i(t)) \quad \forall i \in \{1, 2, \ldots, g\}; \tag{8}$$

i.e., we include copies of the nonlinearities (2) in the controller. Hence, Eqs. (7), (8) define a nonlinear output feedback controller with the property that the functional form of the nonlinearities entering into the controller is the same as the functional form of the nonlinearities entering into the plant. However, the inputs to these controller nonlinearities are not the same as the inputs to the plant nonlinearities. Then, we move the nonlinearities (8) into the plant description and combine the inputs and outputs of the system (7) as follows:

$$\begin{aligned} &\tilde{y} \triangleq [y' \quad \tilde{\mu}_1' \quad \ldots \quad \tilde{\mu}_g']'; \qquad \tilde{u} \triangleq [u' \quad \tilde{\nu}_1' \quad \vdots \quad \tilde{\nu}_g']'; \\ &\tilde{B}_c \triangleq [B_c \quad \tilde{G}_{c1} \quad \ldots \quad \tilde{G}_{cg}]; \qquad \tilde{C}_c \triangleq [C_c' \quad \tilde{K}_{c1}' \quad \ldots \quad \tilde{K}_{cg}']'. \end{aligned} \tag{9}$$

Using this notation, the controller (7) can be re-written

$$dx_c(t) = A_cx_c(t)dt + \tilde{B}_cd\tilde{y}(t); \qquad \tilde{u}(t) = \tilde{C}_cx_c(t) \tag{10}$$

and the problem of controlling the nonlinear uncertain system (1), (2), (4), (5) via the nonlinear controller (7), (8) is equivalent to the problem of controlling the nonlinear uncertain system (1), (2), (4), (5), (8), (with repeated nonlinearities) via the linear controller (10).

2.2. Integral quadratic constraints and dynamic multipliers

It is straightforward to verify that the uncertainties (4), (5) satisfy the IQCs

$$\int_0^\infty [\xi' \quad \zeta']M_i\begin{bmatrix}\xi \\ \zeta\end{bmatrix}dt \geq 0 \quad \forall i = 1, \ldots, k. \tag{11}$$

Here, $M_i = \mathrm{diag}\{0, \ldots, 0, -I, 0, \ldots, 0, I, 0, \ldots, 0\}$ where the matrix $-I$ is the ith block of M_i and the matrix I is the $(k+i)$th block of M_i. Also $\|\cdot\|$ denotes the standard Euclidean norm.

To obtain IQCs and dynamic multipliers for the repeated nonlinearities (2), (8), we use the results of D'Amato et al. (2001). For any $\psi_i(\nu_i)$ satisfying condition (3), we construct a dynamic multiplier defined by a 2×2 symmetric matrix G^i and an LTI system with symmetric 2×2 transfer function matrix $H^i(s)$ and whose impulse response matrix $h^i(\cdot)$ has all of its entries in $\mathcal{L}_1$.