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Brief paper

Robust exponential stabilization for Markovian jump systems with mode-dependent input delay ☆

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Abstract

This paper is concerned with the problem of exponential stabilization for uncertain linear systems with Markovian jump parameters and mode-dependent input delays. Sufficient stabilization conditions are developed in terms of matrix inequalities, which can be solved by a proposed iterative algorithm based on the cone complementarity linearization (CCL) method. Memory controllers are also designed such that the closed-loop system is exponentially mean-square stable for all admissible uncertainties. Numerical examples are given to show that the developed method is efficient and less conservative.

Keywords: Delay system; Exponential stability; Input delay; Markovian jump system; Robust stabilization

1. Introduction

Markovian jump linear systems were firstly introduced by Krasovskii and Lidskii (1961). Since then, a great deal of attention has been devoted to the study of this class of systems in recent years. This class of systems can model different types of dynamic systems subject to abrupt changes in their structures, such as failure prone manufacturing systems, power systems and economics systems, etc. (Boukas, Liu, & Liu, 2001). This advantage in modelling has attracted an increasing interest in the research of such systems, and many results have been reported in the literature; see Boukas (2005), Feng, Loparo, Ji, and Chizeck (1992), Mahmoud, Shi, and Ismail (2004), Mao (2002), Sun, Xu, and Zou (2006), Xu, Chen, and Lam (2003), and references therein.

On the other hand, in practical engineering, time delays are commonly encountered and are often the sources of instability or degradation of control performance (Chen, Lam, & Xu, 2006; Fridman & Shaked, 2003). Recently, much attention has been paid to the investigation of robust stabilization problems for uncertain linear systems with input delay. A stabilizing robust controller is proposed in Cheres, Palmor, and Gutman (1990) based on the reduction method; however, the implementation of the controller is quite complex. Without using the reduction method, memoryless controllers which make use of current state information only are designed and delay-independent stabilization conditions are given in Choi and Chung (1995) and Kim, Jeung, and Park (1996). More recently, a new memoryless state feedback controller for uncertain systems with delays in the control input is designed (Kwon & Park, 2006b) through constructing a new Lyapunov function with free weighting matrices. Although memoryless controllers have relatively simpler structure and are easier to implement, they often lead to much conservatism. To reduce the conservatism, memory controllers are employed in the robust stabilization problem for systems with control input. This kind of controller utilizes information of both the current state and the past input information, which is expected to compensate the effect of the input delay. By using the memory controller, Moon, Park, and Kwon (2001) derived a stabilizing criterion which was shown to be less

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conservative via an example. However, the disadvantage of this method is the exact value of the input delay needs to be known in advance. The method of Moon et al. (2001) was extended in Yue (2004) and Yue and Han (2005) to give new robust stabilization criteria for uncertain linear systems with constant and time-varying input delay, respectively. It is worth pointing out that the result of Yue (2004) can be applied to the case in which the input delay is not exactly known in advance. The drawback of this approach is that it involves many computation variables and some scalars are required to be specified in advance, which limit its application or lead to less convenience.

This paper considers the robust exponential stabilization problem of uncertain Markovian jump linear systems with input delay. The parameter uncertainties are assumed to be time-varying but norm-bounded, and the input delay is mode-dependent and is a known constant for each mode. Through constructing a Lyapunov functional, sufficient conditions are derived in terms of matrix inequalities to guarantee the jump system to be robustly stochastically stabilizable. The corresponding memory controllers are then designed. An algorithm based on the cone complementarity linearization (CCL) method (El Ghaoui, Oustry, & Aitrami, 1997) is proposed to solve the matrix inequalities. Finally, several numerical examples are given to show the effectiveness and the reduced conservatism of our approach.

Notation: For real symmetric matrices P, Q, $P > Q$ (respectively, $P \geqslant Q$) denotes that $P - Q$ is a positive definite (respectively, positive semi-definite) matrix. The notation $\|\cdot\|$ refers to the spectral norm for matrices, $|\cdot|$ refers to the Euclidean norm for vectors, and $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue. Moreover, $\mathcal{E}(\cdot)$ stands for the mathematical expectation operator, and $\max\{\cdot\}$, $\min\{\cdot\}$ denote the maximum and minimum values, respectively.

2. Problem formulation

Consider the following uncertain system with Markovian jump parameters and input delay (Σ):

$$\begin{aligned}\dot{x}(t) = {} & [A(r(t)) + \Delta A(t, r(t))] \\ & x(t) + [B(r(t)) + \Delta B(t, r(t))]u(t) \\ & + [C(r(t)) + \Delta C(t, r(t))]u(t - h(r(t))),\end{aligned} \tag{1}$$

$$x(0) = x_0, \quad r(0) = r_0, \quad u(t) = \phi(t), \quad \forall t \in [-h, 0], \tag{2}$$

where $x(t) \in \mathbb{R}^n$ stands for the state of the system, $u(t) \in \mathbb{R}^m$ is the control input vector, $\{x_0, r_0, \phi(t)\}$ is the initial condition. The parameter $r(t)$ represents a continuous-time Markov process on the probability space, which takes values in a finite set $S = \{1, 2, \dots, N\}$ with transition probabilities given by

$$\Pr\{r(t+\varDelta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\varDelta + o(\varDelta) & \text{if } i \neq j, \\ 1 + \pi_{ii}\varDelta + o(\varDelta) & \text{if } i = j, \end{cases} \tag{3}$$

where $\varDelta > 0$ and π_{ij} stands for the transition probability rate from mode i to mode j when $i \neq j$ and satisfies the following relations for all $i \in S$:

$$\pi_{ij} \geqslant 0 \quad \text{and} \quad \pi_{ii} = -\sum_{j \in S, j \neq i} \pi_{ij}. \tag{4}$$

For brevity, we denote $A_i = A(r(t))$ and $\Delta A_i(t) = \Delta A(t, r(t))$ for each $r(t) = i \in S$, and the other symbols are similarly denoted. It is assumed that, for each $r(t) = i \in S$,

$$\begin{aligned}\Delta A_i(t) &= D_i F_i(t) E_i, \\ \Delta B_i(i) &= \bar{D}_i \bar{F}_i(t) \bar{E}_i, \\ \Delta C_i(t) &= \hat{D}_i \hat{F}_i(t) \hat{E}_i,\end{aligned} \tag{5}$$

where $F_i(t)$, $\bar{F}_i(t)$, $\hat{F}_i(t)$ are unknown real matrices satisfying

$$F_i^{\mathrm{T}}(t) F_i(t) \leqslant I_p, \quad \bar{F}_i^{\mathrm{T}}(t) \bar{F}_i(t) \leqslant I_q,$$

$$\hat{F}_i^{\mathrm{T}}(t) \hat{F}_i(t) \leqslant I_r, \quad t > 0.$$

The uncertainty matrices are said to be admissible if (5) holds.

In this note, the system state and the system mode are assumed to be accessible and the input delay h_i is assumed to be a known constant for each $r(t) = i \in S$ and $h = \max_{i \in S}\{h_i\}$.

Definition 1 (*Feng et al., 1992*). The nominal Markovian jump system of (Σ) (with $u(t) \equiv 0$) is said to be exponentially mean-square stable (EMSS) if

$$\mathcal{E}[|x(t)|^2] \leqslant \alpha |x_0|^2 \mathrm{e}^{-\beta t}, \tag{6}$$

for any finite $x_0 \in \mathbb{R}^n$ and $r_0 \in S$, where $x(t)$ is the trajectory of the system state from initial system state x_0 and initial mode r_0, and α, β are positive constants.

Definition 2. System (Σ) is said to be robustly stochastically stabilizable if there exists a state feedback controller such that the state trajectory of the closed-loop system satisfies (6) for all admissible uncertainties.

This paper is concerned with the robust stochastic stabilization problem for uncertain Markovian jump systems with input delay. The purpose is to find sufficient conditions for robust stochastic stabilization of the system and to design a corresponding state feedback controller.

3. Main results

In this section, we will develop sufficient conditions for the robust stochastic stabilization for the uncertain jump system with input delay and design memory controllers in the following form:

$$u(t) = K(r(t)) \left[x(t) + C(r(t)) \int_{t-h(r(t))}^{t} u(s)\, \mathrm{d}s \right], \tag{7}$$

when there is only one mode, that is, the Markovian jump linear system reduces to an ordinary linear system without jump, the controller above becomes $u(t) = K[x(t) + \int_{t-h}^{t} Cu(s)\, \mathrm{d}s]$, which has been employed in many stabilizing problems especially for