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Brief paper

A globally stable saturated desired compensation adaptive robust control for linear motor systems with comparative experiments☆

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Abstract

The recently proposed saturated adaptive robust controller is integrated with desired trajectory compensation to achieve global stability with much improved tracking performance. The algorithm is tested on a linear motor drive system which has limited control effort and is subject to parametric uncertainties, unmodeled nonlinearities, and external disturbances. Global stability is achieved by employing back-stepping design with bounded (virtual) control input in each step. A guaranteed transient performance and final tracking accuracy is achieved by incorporating the well-developed adaptive robust controller with effective parameter identifier. Signal noise that affects the adaptation function is alleviated by replacing the noisy velocity signal with the cleaner position feedback. Furthermore, asymptotic output tracking can be achieved when only parametric uncertainties are present.

Keywords: Input saturation; Adaptive control; Robust control; Linear motors; Motion control; Nonlinear systems

1. Introduction

Significant research has been devoted to the globally stable controls with input saturation due to the unavoidable actuator saturation for any physical devices (Bernstein & Michel, 1995). Research has been done that focuses on either the ad-hoc technique of anti-windup or system stabilization via account of the saturation nonlinearities at the controller design stage. All the studies in Sussmann and Yang (1991), Lin and Saberi (1995), Teel (1992, 1995) assumed that the systems under investigation are linear and the system parameters are all known, which is not true for most physical systems. Nonlinear factors such as friction and hysteresis affect system behavior significantly and are rather difficult to model precisely. It is not unusual that some of the system parameters are unknown or have variable values. Unpredictable external disturbances also affect the system performance. Therefore, it is of practical importance to take these issues into account when solving the actuator saturation problem. Gong and Yao (2000) combined the saturation functions proposed in Teel (1992) with the adaptive robust control (ARC) strategy proposed in Yao and Tomizuka (1997), Yao (1997) and achieved global stability and good performance for a chain of integrators subject to matched model uncertainties. Recently, a new saturated control structure was introduced in Hong and Yao (2005). This new scheme is based on backstepping design (Krstić, Kanellakopoulos, & Kokotovic, 1995) and ARC strategy Yao (1997), Yao and Tomizuka (1997). The control law is designed to ensure fast error convergence during normal working conditions while globally stabilizing the system for a much larger class of modeling uncertainties than those considered in Gong and Yao (2000).

The saturated ARC proposed in Hong and Yao (2005) has been applied to a linear motor positioning system. As revealed

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in Yao (1998), some implementation problems were encountered. For example, a noisy velocity measurement may restrict the achievable performance due to the state-dependent regressor. In this paper, the desired trajectory replaces the actual state in the regressor for both model compensation and parameter adaptation in order to further improve the tracking performance while preserving global stability. Furthermore, by using the "integration by parts" technique (Xu & Yao, 2001), the resulting parameter estimation algorithm uses only the feedback position signal, which has micrometer resolution and much less noise contamination than the velocity signal used in Hong and Yao (2005). Comparative experimental results show that the tracking error is reduced almost by half.

2. Problem formulation

The essential dynamics of the linear motor system detailed in Xu and Yao (2001) are

$$M\ddot{y} = K_f u - B\dot{y} - F_{sc}S_f(\dot{y}) + d(t), \tag{1}$$

where M is the inertia of the payload plus the coil assembly, y is the stage position, u is the control voltage with an input gain of K_f, B and F_{sc} represent the two major friction coefficients, viscous and Coulomb, respectively, $S_f(\dot{y})$ is a differentiable with uniformly bounded derivatives, non-decreasing function that approximates the discontinuous sign function $\mathrm{sgn}(\dot{y})$ which is normally used in the modeling of Coulomb friction as in Xu and Yao (2001), $d(t)$ represents the lumped friction modeling error, other unmodeled dynamics (e.g., the force ripples of linear motors), and external disturbances.

The above system is subject to unknown parameters due to payload variation, uncertain friction coefficients, and lumped neglected model dynamics and external disturbances. In this paper, the mass term M is assumed to be known since, compared to other terms, it is unlikely to change once the payload is fixed and easy to estimate accurately either off-line or on-line; it is nevertheless noted that the method presented below can be extended to the case when M is unknown without much theoretical difficult. The low frequency component of the lumped uncertainties is modeled as an unknown constant. Therefore, letting $\mathbf{x} = [x_1, x_2]^{\mathrm{T}} := [y, \dot{y}]^{\mathrm{T}}$ be the state vector, the above governing dynamic equations can be rewritten in a state-space form as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -B_m x_2 - F_{scm}S_f(x_2) + d_{0m} + \Delta + \frac{K_f}{M}u \\ &= \boldsymbol{\varphi}^{\mathrm{T}}(\mathbf{x})\boldsymbol{\theta} + \Delta + \bar{u}, \end{aligned} \tag{2}$$

where $\boldsymbol{\varphi}(\mathbf{x}) = [-x_2, -S_f(x_2), 1]^{\mathrm{T}}$ is the regressor, $\boldsymbol{\theta} = [B_m, F_{scm}, d_{0m}]^{\mathrm{T}} = [B, F_{sc}, d_0]^{\mathrm{T}}/M$ is the vector of unknown parameters to be adapted on-line, in which d_{0m} represents the nominal value of the normalized disturbance $d_m(t) = d(t)/M$, $\Delta = d_m(t) - d_{0m}$ represents the variation or high frequency components of $d_m(t)$, and $\bar{u} = K_f u/M$ is the normalized control input whose upper and lower limits can be calculated from the physical input saturation level. The following practical assumptions are made:

Assumption 1. The extents of the parametric uncertainties are known, i.e.,

$$B_m \in [B_{ml}, B_{mu}], \quad F_{scm} \in [F_{scml}, F_{scmu}],$$

where B_m, F_{scm} are positive according to the real system and B_{ml}, B_{mu}, F_{scml}, F_{scmu} are known.

Assumption 2. The lumped disturbance $d_m(t)$ is bounded, i.e.,

$$|d_m(t)| \leqslant \delta_{dm},$$

where δ_{dm} is known.

The desired trajectory, position x_{1d}, velocity $x_{2d} = \dot{x}_{1d}$ and acceleration $\ddot{x}_{1d}$, are assumed to be known and bounded. Let $\bar{u}_{bd}$ represent the normalized bound of the actuator authority. *The saturation control problem can be stated as: under the above assumptions and the normalized input constraint of $|\bar{u}(t)| \leqslant \bar{u}_{bd}$, design a control law that globally stabilizes the system and makes the output tracking error $z_1 = x_1 - x_{1d}(t)$ as small as possible.*

3. Saturated desired compensation ARC

3.1. Controller structure

From Assumptions 1 and 2, it is obvious that the parameter vector $\boldsymbol{\theta}$ belongs to a set Ω_θ as $\boldsymbol{\theta} \in \Omega_\theta = \{\boldsymbol{\theta}_{\min} \leqslant \boldsymbol{\theta} \leqslant \boldsymbol{\theta}_{\max}\}$, where $\boldsymbol{\theta}_{\min} = [B_{ml}, F_{scml}, -\delta_{dm}]^{\mathrm{T}}$, $\boldsymbol{\theta}_{\max} = [B_{mu}, F_{scmu}, \delta_{dm}]^{\mathrm{T}}$, and the operation $\leqslant$ for two vectors is performed in terms of their corresponding elements. Let $\hat{\boldsymbol{\theta}}$ denote the estimate of $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{\theta}}$ be the estimation error $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$. The following projection-type parameter adaptation law (Yao & Tomizuka, 1997) is used

$$\dot{\hat{\boldsymbol{\theta}}} = Proj_{\hat{\theta}}(\Gamma\boldsymbol{\tau}), \quad \hat{\boldsymbol{\theta}}(0) \in \Omega_\theta, \tag{3}$$

$$Proj_{\hat{\theta}}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{i\max} \text{ and } \bullet_i > 0, \\ 0 & \text{if } \hat{\theta}_i = \theta_{i\min} \text{ and } \bullet_i < 0, \\ \bullet_i & \text{otherwise}, \end{cases} \tag{4}$$

where Γ is a diagonal matrix of adaptation rates and $\boldsymbol{\tau}$ is an adaptation function to be synthesized further on. Such a parameter adaptation law has the following desirable properties (Yao, 1997).

(P1) The parameter estimates are always within the known bound at any time instant, i.e., $\hat{\boldsymbol{\theta}}(t) \in \Omega_\theta$.

(P2) $\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\boldsymbol{\tau}) - \boldsymbol{\tau}) \leqslant 0, \forall \boldsymbol{\tau}$.

The controller design follows the same back-stepping procedure as in Hong and Yao (2005). Define $z_1 = x_1 - x_{1d}$ as the tracking error, α_1 as the bounded virtual control law designed for z_1 dynamics, which is $\dot{z}_1 = x_2 - \dot{x}_{1d}$. Define $z_2 = x_2 - \alpha_1$, then z_1 dynamics become

$$\dot{z}_1 = z_2 + \alpha_1 - \dot{x}_{1d}. \tag{5}$$