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Brief paper

Tradeoffs between quantization and packet loss in networked control of linear systems☆

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ABSTRACT

In this paper, we consider to derive the coarsest memoryless quantizer which can stabilize a single-input discrete-time linear time-invariant system with stochastic packet loss in the sense of stochastic quadratic stability. We show that the upper bound of the coarseness is strictly given by the packet loss probability and the unstable poles of the plants. We furthermore deal with permissible dead-zone width around the origin of the quantizers and time-varying finite quantizers in order to realize control using finite quantization steps.

1. Introduction

In recent years, networked control systems have been actively investigated in the field of control theory and one of the interests is to find the relationship between the permissible coarseness of transmitted signals for stabilization and the properties of plants. Some of the recent works on this topic include (Brockett & Liberzon, 2000; Elia & Mitter, 2001; Fu & Xie, 2005; Goodwin, Haimovich, Quevedo, & Welsh, 2004; Nair & Evans, 2004; Tatikonda & Mitter, 2004a; Tsumura & Maciejowski, 2003; Wong & Brockett, 1999). In particular, in Elia and Mitter (2001) a stabilization problem via quantized input signals is considered and the coarsest memoryless quantizer for stabilization of single-input discrete-time linear time-invariant systems is derived. A notable point is that the upper bound of the coarseness is given only by the unstable poles of the plants. This result is also extended to LQR type problems (Fu & Xie, 2005) and adaptive control problems (Hayakawa, Ishii, & Tsumura, 2009a,b).

Another problem we should deal with for networked control systems is the packet loss in data transmission. This problem arises when unreliable communication channels are used such as wireless networks or general-purpose channels. Clearly, losses of signals cause performance degradation or can make a closed-loop system unstable. Some research groups have dealt with this problem. LQ type control problems are considered in Imer, Yüksel, and Başar (2006), and H_∞ control approaches were proposed in Seiler and Sengupta (2005) and Ishii (2008a). Sinopoli et al. (2004), studied stabilization in state estimation problems under packet losses. In Elia (2005) and Ishii (2008b), the mean square stability of feedback control systems is investigated and the upper limit of loss probability is given in terms of the unstable poles of the plants. For the scalar case, this was shown in Hadjicostis and Touri (2002).

In spite of the above significant results showing the relationships between "the unstable poles of plants and the coarseness of quantization (Elia & Mitter, 2001)" and between "the unstable poles of plants and the packet loss probability (Elia, 2005; Ishii, 2008b)," in real communication channels, it is more realistic to assume that the channel contains both quantization and stochastic packet losses. A natural extension of our interests is on the relationship among the three properties above for such networked control systems.

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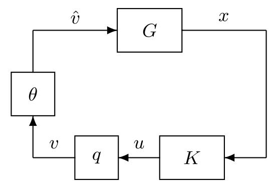

Fig. 1. Stabilization via quantized signals with stochastic packet losses.

This is our motivation of research and we investigate the coarsest memoryless quantizer for stabilization with stochastic packet losses. We show in particular that this upper bound of the coarseness is strictly given by the packet loss probability and the unstable poles of the plants (Section 2). As a consequence, we integrate and generalize the previous results of Elia and Mitter (2001) and those of Elia (2005) and Ishii (2008b).

In this paper, we furthermore deal with permissible dead-zone width around the origin of the quantizer for a stochastic version of practical stability (Section 3) and time-varying finite quantizers (Section 4) for realizing realistic quantizers which have finite quantization steps.

2. The coarsest quantizer for stabilization with stochastic packet losses

In this paper, we consider the following discrete-time linear system:

$$G:\ x(k+1) = Ax(k) + B\hat{v}(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $\hat{v}(k) \in \mathbb{R}$ is the control input, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times 1}$. Assume that (A, B) is stabilizable and A is unstable.

We explain how the control signal is processed when it is transmitted from a controller to the control input of G according to Fig. 1. At first, the control signal $u(k)$ from the controller is quantized at the controller side before it is sent over a communication channel. The quantization is given by

$$v(k) = q(u(k)), \tag{2}$$

where $q(\cdot)$ is a memoryless quantizer and $u(k) \in \mathbb{R}$ is an ordinary analog control input generated by a static state feedback controller $K(\cdot)$.

In addition, we assume that packet losses occur with probability α at the input-side channel of the plant. In this paper, we employ a simple scheme where the packet loss sets $\hat{v}(k) = 0$,[1] and hence the system can be described as

$$x(k+1) = Ax(k) + B\theta(k)v(k), \tag{3}$$

where $\theta(k)$ is a 0–1 random variable with a probability distribution given by

$$\Pr(\theta(k) = i) = \begin{cases} \alpha, & i = 0, \\ 1-\alpha, & i = 1, \end{cases} \quad 0 \le \alpha < 1.$$

The reason why we deal with the case that the quantization is limited to the plant input side is that it is one of the basic setups. It is also a model where a large difference exists in the capacities for transmissions to and from the controller such as in a wireless-networked control system or a large-scale plant.

We next describe the stability we employ in this section. Consider the following discrete-time system:

$$x(k+1) = f(x(k), \theta(k)), \tag{4}$$

where $x(k) \in \mathbb{R}^n$ is the state, and $\theta(k) \in \{0, \ldots, N-1\}$ represents the mode of the system. The mode is an independent and identically distributed stochastic process with probabilities $\alpha_i = \Pr(\theta(k) = i)$. The function $f(x, \theta)$ satisfies $f(0, \theta) = 0$ for arbitrary θ. Thus, the origin $x = 0$ of the system is an equilibrium point.

For this system we define the following stability:

Definition 2.1. For the system (4), the equilibrium point at the origin is stochastically quadratically stable if there exists a positive-definite function $V(x) = x^{\mathrm{T}}Px$ and a positive-definite matrix R such that

$$\begin{aligned}\Delta V &= E[V(x(k+1))|x(k)] - V(x(k)) \\ &\le -x(k)^{\mathrm{T}}Rx(k), \quad \forall x(k) \in \mathbb{R}^n.\end{aligned} \tag{5}$$

Remark 2.1. The condition (5) is sufficient for the origin of the system (4) to be mean square stable (see, e.g., Ji and Chizeck (1990)), i.e., for every initial state x_0,

$$\lim_{k\to\infty} E[\|x(k)\|^2|x_0] = 0. \tag{6}$$

The important point on the condition (5) is that the absolute averaged decreasing rate of a Lyapunov function V is larger than or equal to a quadratic form of x. Also we should note that another condition $\Delta V < 0, \forall x$, does not necessarily guarantee stability for the "stochastic" nonlinear system different from the case Elia and Mitter (2001). The matrix $R\,(> O)$ in (5) regulates the convergence rate of x and it is critical for the moment of x as shown in Proposition 3.1 and Theorem 3.1, Section 3, where we deal with a case that the quantizer has a dead-zone.

In this section, our objective is to find the coarsest quantizer $q(\cdot)$ which achieves stochastic quadratic stability for the system (3). The coarseness of a quantizer $q(\cdot)$ is defined as (Elia & Mitter, 2001)

$$d = \limsup_{\epsilon\to 0} \frac{\sharp u[\epsilon]}{-\ln \epsilon}, \tag{7}$$

where $\sharp u[\epsilon]$ denotes the number of levels that the quantizer $q(\cdot)$ has in the interval $[\epsilon, 1/\epsilon]$.

Elia and Mitter (2001) showed that the coarsest quantizer for the quadratic stabilization in the case of *no packet loss* is logarithmic and the coarsest expansion ratio ρ_{sup} (which is strictly defined later) is given by

$$\rho_{\mathrm{sup}} = \frac{\prod_i |\lambda_i^u| + 1}{\prod_i |\lambda_i^u| - 1}, \tag{8}$$

where λ_i^u represents the unstable poles of the plant. On the other hand, in Elia (2005) and Ishii (2008b), a necessary and sufficient condition on α for the mean square stabilizability in the case of *no quantization* is given as

$$\alpha < \alpha_{\mathrm{sup}} = \frac{1}{\prod_i |\lambda_i^u|^2}. \tag{9}$$

In this paper, we consider the effects of both *quantization and packet losses* and the natural extension of our interests is "What relationship between ρ_{sup}, α and λ_i^u does there exist?" We provide a complete answer to this question in the following theorem, which unifies the results (8) and (9).

Theorem 2.1. *The coarsest quantizer $q_c(\cdot)$ with which the system (3) is stochastically quadratically stable is given as:*

$$q_c(u) = \begin{cases} v_i, & u \in \left(\dfrac{\rho_{\mathrm{sup}}+1}{2\rho_{\mathrm{sup}}}v_i, \dfrac{\rho_{\mathrm{sup}}+1}{2}v_i\right], \\ -v_i, & u \in \left[-\dfrac{\rho_{\mathrm{sup}}+1}{2}v_i, -\dfrac{\rho_{\mathrm{sup}}+1}{2\rho_{\mathrm{sup}}}v_i\right), \\ 0, & u = 0, \end{cases} \tag{10}$$

[1] More complex models for packet loss are possible; however, we deal with the simple and standard model in this paper.