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Effect of viscous force on dynamic contact angle measurement using Wilhelmy plate method

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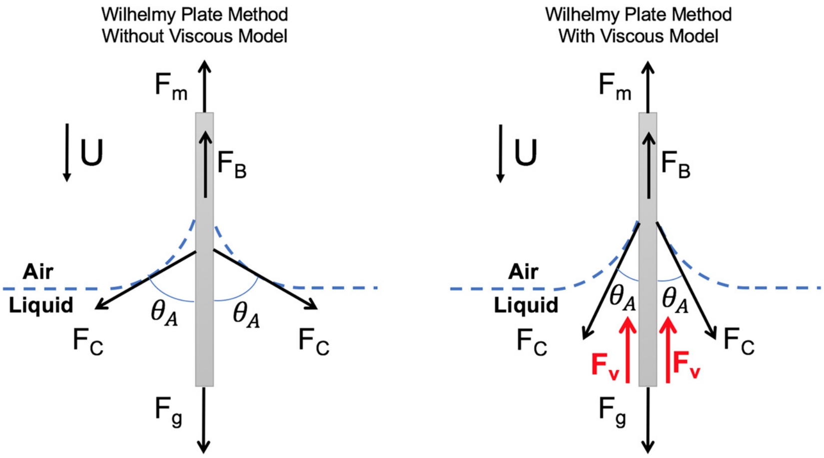
GRAPHICAL ABSTRACT



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ABSTRACT

Wilhelmy plate method does not consider viscous force in the force balance equation to measure the dynamic contact angle and this results in a significant error in the measurement. Differences between the results obtained by optical method and Wilhelmy plate method indicate the importance of viscous force in the force balance equation. A theoretical viscous model is proposed, which must be considered in the force balance equation in Wilhelmy plate method to increase the accuracy of the dynamic contact angle measurement, especially for the case of highly viscous liquids and for experiments at large speeds of the plate, which can lead to large shear rate along the surface of the plate.

1. Introduction

The Wilhelmy plate method is extensively used to study the static and dynamic contact angles [1–3] such as contact angle hysteresis on a solid surface [4–9] and contact line pinning [10]. In the Wilhelmy plate method, a plate is immersed into or withdrawn out of a pool of liquid with a certain speed and the dynamic contact angle is measured during the plate's motion. The Wilhelmy plate method considers the capillary force, F_C, due to surface tension of the liquid, the buoyancy force, F_B, the weight of the plate, F_g, and the detected plate holder force by microbalance force sensor, F_m, to keep the plate moving with constant speed during its immersion/withdrawal into or withdrawn out of the

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liquid pool. Since the thickness of the plate is very small, the form drag (i.e. pressure drag) is negligible. This method does not involve a direct measurement of the dynamic contact angle, but determines the dynamic contact angle using a force balance Eq. (1) [11–14]:

$$F_m + F_C + F_B + F_g = 0. \tag{1}$$

The capillary force, the buoyancy force, and the weight of the plate are defined by the physical properties of the liquid and the geometric specifications of the plate (Eqs. (2a)–(2c)):

$$F_C = 2\sigma\,(w + t)\,\cos\theta, \tag{2a}$$

$$F_B = \rho\,g\,w\,t\,x, \tag{2b}$$

$$F_g = \rho_s\,g\,w\,t\,l, \tag{2c}$$

where σ is the surface tension, ρ is the liquid density, w, t and l are width, thickness, and length of the plate, respectively, θ is the dynamic contact angle, ρ_s is the plate density, g is the gravitational acceleration, and x is the immersion depth (i.e. the length of the plate below the surface of the liquid pool).

Fig. 1 presents a schematic of the force balance diagram that is used in the tensiometer software using the Wilhelmy plate method to measure the advancing dynamic contact angle as the liquid contact line (i.e. solid-liquid-air interface line) moves due to immersion of the plate into the liquid pool.

Due to the theoretically based measurement of the dynamic contact angle with a highly sensitive microbalance force sensor, the Wihelmy plate method is expected to produce more accurate and highly reproducible results compared to other experimental methods [15–19]. However, Eq. (1) used in the Wilhelmy plate method does not consider the viscous force in the measurement which can result in errors in the dynamic contact angle measurement. These errors can be neglected for cases with low-viscosity liquids and low-speed of the plate. Thus, the speed of the plate is limited to low values in order to obtain accurate results. This limitation can cause the Wilhelmy plate method to lose its extent of applicability for a wider range of experimental studies. Thus, it is important to understand the effect of the ignored viscous force on the dynamic contact angle measurement and to correct for it.

In this study, a new force balance equation which includes the viscous force on the plate is proposed to measure the dynamic contact angle at higher speeds and for more viscous liquids. A series of experiments with Newtonian liquids over a large range of viscosity were done to quantitatively understand the effect of viscous force on the dynamic contact angle measurement.

2. Materials and methods

2.1. Instrumentation

A K100 tensiometer (Krüss GmbH, Hamburg, Germany) based on the Wilhelmy plate method was used in the experiments reported here. The plate is connected to the microbalance force sensor with a clamp. The liquid container on the platform (i.e. container holder) moved upward at a specified constant speed to immerse the plate into the liquid pool. During the immersion of the plate into the liquid pool, the microbalance force sensor measures the force applied on the moving plate that is recorded by built in software in the tensiometer. Algorithms based on the Wilhelmy plate method used force balance (Eq. (1)) to calculate the advancing dynamic contact angle. The speed of the platform was set to a constant value to maintain a steady motion during the experiment. Fig. 2 shows a schematic picture and image of the tensiometer equipment.

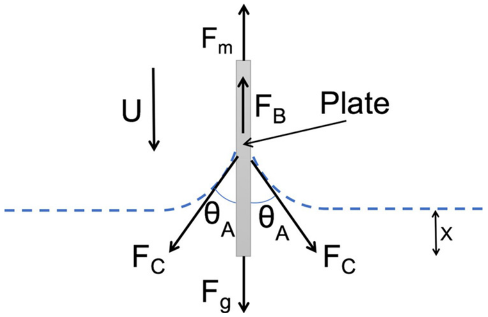

Fig. 1. Schematic of forces applied on a plate during immersion of the plate in a liquid pool.

In order to verify the accuracy of the dynamic contact angles calculated using the Wilhelmy plate method, an optical method was also designed and applied simultaneously during the experiment with the tensiometer to record images of the process of the plate's immersion into the liquid pool. Fig. 2 shows a schematic of the optical system used to measure the dynamic contact angle during the experiment with the tensiometer. The optical system consisted of a Canon ultrasonic EOS-1 camera with a single lens reflex SLR. The camera was focused on the liquid contact line (i.e. plate-liquid-air interface line) and the images captured the menisci formed at the liquid contact line during the motion of the plate into the liquid pool. The advancing dynamic contact angles were measured using ImageJ [20].

2.2. Materials

Three Newtonian liquids were used to cover a wide range of viscosities (from 96 to 1412 mPa s) for the dynamic contact angle measurements using the Wilhelmy plate method and an optical method. Table 1 shows the physical properties of the liquids used in the experiments reported here. The test solid substrates are thin glass plates. Glass plates with dimensions of $50 \times 24 \times 0.15\ \mathrm{mm}^3$ (VWR microcover glass) were rinsed in DI water. After being cleaned, each plate was dried with compressed air before the experiments.

3. Theory

As a solid plate moves into a pool of viscous liquid, a thin viscous boundary layer is formed along the surface of the plate. A viscous model should be added to Eq. (1) for precise dynamic contact angle measurements with the Wilhelmy plate method. Fig. 3 represents schematic of the viscous boundary layer formed on the solid flat plate moving into a pool of viscous liquid.

A model that signifies the effects of the viscous force on the plate during force calculation has been obtained for the case of low Reynolds number motion of a moving solid plate into a pool of liquid by applying boundary layer theory [12]. The assumptions of steady state flow, incompressible fluid, and creeping flow have been applied in the analysis [12].

A viscous model analysis was carried out for the situation of two-dimensional flow of a viscous liquid near a corner of a solid plate with an angle of the corner to be 180°. This problem is similar to the Taylor problem [21]. The solid flat plate has width, w, thickness, t, and length, l. In this case, the solid flat plate is being immersed with a constant speed, U, into a pool of viscous liquid through the interface of the liquid/air as illustrated in Fig. 4. In the polar-coordinate system shown in Fig. 4, the velocity in the radial direction, r, is u_r, and the velocity in the θ direction is u_θ.

The following boundary conditions are applied to this problem in a polar coordinate system which are no slip condition along the surface of the solid plate (Eqs. (3a) and (3b)), no shear stress along the liquid-air interface (Eq. (3c)), and no flow-through condition along the liquid-air interface (Eq. (3d)):

$$u_r = U,\quad \theta = -\alpha = -\frac{\pi}{2} \tag{3a}$$