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Brief paper

Multi-rate stochastic H_∞ filtering for networked multi-sensor fusion[☆]

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ABSTRACT

This paper presents a multi-rate filtering problem for a class of networked multi-sensor fusion systems with packet dropouts (PDs): the state evolves according to a linear discrete-time model with norm-bounded unknown inputs (UIs), and its underlying period is h; the p sensors are distributively deployed with different sampling periods $n_1h, \dots, n_ph$; multi-rate sensor measurements, corrupted by UIs, are subject to stochastic PDs in the transmission to a fusion center for state estimation; the estimation is updated at the period mh. Different from the single-rate estimator design with PDs which are treated as stochastic parameters, a UI observer is proposed where PDs are represented as zero-mean white input noises of the linear time-variant estimation error system. The results on the existence of a stable observer are proposed. Due to insufficient design freedom for absolute error decoupling, we turn to designing an observer-based stochastic H_∞ filter. A numerical example of distributive multi-sensor target tracking is given to illustrate the proposed filter.

1. Introduction

Networked control systems (NCSs) have gained attention during the last few years (e.g., see Basin & Martinez-Zuniga, 2004; Hespanha, Naghshtabrizi, & Xu, 2007; Hu & Zhu, 2003; Luck & Ray, 1990; Matveev & Savkin, 2003; Nilsson, Bernhardsson, & Wittenmark, 1998; Wang, Yang, Ho, & Liu, 2006; Zhang, Basin, & Skliar, 2007, and references therein). When compared with using the conventional point-to-point system connection, using an NCS has advantages like easy installation and reduced setup, wiring, and maintenance costs. In an NCS, data travel through the communication channels from sensors to the controller and/or from the controller to the actuators. For the estimation problem, an immediate concern arising from the introduction of these network media is that the network properties, such as random transmission delay and packet dropout (PD), give rise to parameter uncertainties.

Most research conducted on NCSs considers the issues of random transmission delay and uncertain observations where the associated systems can be described by stochastic systems (see e.g., Basin & Martinez-Zuniga, 2004; Nilsson et al., 1998; Wang et al., 2006; Zhang et al., 2007, and references therein). The closely related random PD has been the focus of some research studies in the last few years. The stability of a time-varying Kalman filter in relation to a PD rate was investigated (Sinopoli, Schenato, Franceschetti, Jordan, & Sastry, 2004). Through transforming the PD rate into a stochastic parameter in the system's representation, stochastic H_2 and H_∞ norms of an estimation error system were defined, and thus H_2 and H_∞ filters were proposed to deal with the possible delay of one sampling period, uncertain observations and multiple PDs under a unified framework (Sahebsara, Chen, & Shah, 2007, 2008). Considering a linear system with stochastic parameters due to PDs, Sun, Xie, Xiao, and Soh (2008a) proposed the optimal linear minimum-variance (LMV) estimators, including filters, predictors and smoothers via innovation analysis approaches. Furthermore, a reduced-order filter was designed (Sun, Xie, Xiao, & Soh, 2008b) and the maximum successive PD number was considered (Sun, Xie, Xiao, & Soh, 2008c). In Gao and Chen (2007), a robust H_∞ estimation scheme for a class of linear systems with polytypic uncertain parameters was studied subject to limited communication capacity; here, as the main properties of limited communication channels, measurement quantization, data transmission delay, and PDs were considered simultaneously in the estimator design. One common step of the above methods is to augment the state using the transmitted measurement and/or received input to capture the dynamic properties introduced by PDs, and then design filters for linear systems with stochastic parameters.

In many complex systems, it is often unrealistic or sometimes impossible to guarantee all physical signals operating at one single rate (Chen & Francis, 1995). State estimation with dual-rate

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sensors was proposed in Andiusani and Gau (1987). Its dynamic system was decomposed into dual subsystems corresponding to the dual-rate sensors; the filtering residuals of one Kalman filter for the fast-rate subsystem were fused with slow-rate sensor measurements by another Kalman filter based on the slow-rate subsystem. Since different dynamic models always have different frequency properties in multiple model systems, the fast-rate sensor measurement was thus compressed to be a slow-rate substitution with little or no accuracy degradation in low-frequency models, and cost-effective multi-rate interacting multiple model estimators were proposed (Hong, 1999). In the case that the updating rate of state estimates is a multiple of the measurement sampling rate, fast-rate estimation was proposed by extending traditional single-rate H_2 or H_∞ filters (Sheng, Chen, & Shah, 2005). In fast-rate fault detection for sampled-data systems with slow-rate measurements, fast-rate residuals were generated (Izadi, Zhao, & Chen, 2005; Zhong, Ye, Ding, & Wang, 2007). Recently, we proposed the LMV estimator for a four-rate estimation problem, which includes the state updating rate in the model, the measurement sampling rate, the estimate updating rate, and the estimate output rate (Liang, Chen, & Pan, 2009). In general, all the above multi-rate filters cannot deal with stochastic time-varying parameters. Thus, designing a multi-rate estimator under stochastic PDs is not trivial by the fact that state augmentation, as the common step in the existing single-rate estimators with PDs, inevitably leads to stochastic parameters.

Here, we present the multi-rate filtering problem for multi-sensor fusion with PDs: sensors are distributively deployed and their sampling rates are not identical; unknown inputs (UIs) drive the evolvement of the interested state and corrupt sensor measurements; multi-rate sensor measurements are subject to PDs in the transmission to the fusion center for state estimation. To the best of the authors' knowledge, this is the first consideration of multi-rate filtering with PDs. Instead along the idea of filter design with stochastic parameters, the problem of multi-rate sensor fusion is transformed into the design of an unknown input observer (UIO). It is interesting that the resultant estimation error system of the observer is a linear time-invariant system with deterministic parameters, and stochastic PDs are presented as the zero-mean white input noises. The conditions for the existence of a stable UIO are proposed. It is founded that such conditions are dependent on the system matrix, measurement matrix, sampling rate of each sensor, and estimation rate, but independent of the non-zero probability of successful packet transmission.

Based on the resultant UIO, we design a multi-rate stochastic H_∞ estimator. A numerical example of distributive multi-sensor target tracking is illustrated. It is worth mentioning that the multi-rate nature gives rise to higher computation complexity in two aspects: first, the lifting process, a common step in treating multi-rate systems, leads to higher dimensional parameter matrices to be optimized; second, the resultant estimator design is non-convex, and thus requires a computation-intensive iterative parameter optimization. Fortunately, such computational issues would not affect the estimator implementation because all estimator parameters are time-invariant and can be optimized off-line.

The rest of this paper is organized as follows. The problem under investigation is formulated in Section 2. The UIO is designed and analyzed in Section 3. Based on the resultant UIO, the multi-rate stochastic H_∞ filter is developed in Section 4. A simulation example is given in Section 5 and some conclusions are drawn in Section 6.

Throughout this paper, the superscripts "-1" and "T" represent the inverse and transpose operations, respectively; the symbols "I" and "0", respectively, represent identity and zero matrices with proper dimensions; diag$\{\cdot\}$ denotes a block diagonal matrix; prob$\{\cdot\}$ denotes a probability measure; $\mathcal{E}\{\cdot\}$ denotes the operator of mathematical expectation; trace$\{\cdot\}$ denotes the operator of matrix trace; $\downarrow m$ represents down-sampler by m; q^{-1} is the discrete-time unit delay operator; rank$\{\cdot\}$ represents the matrix rank.

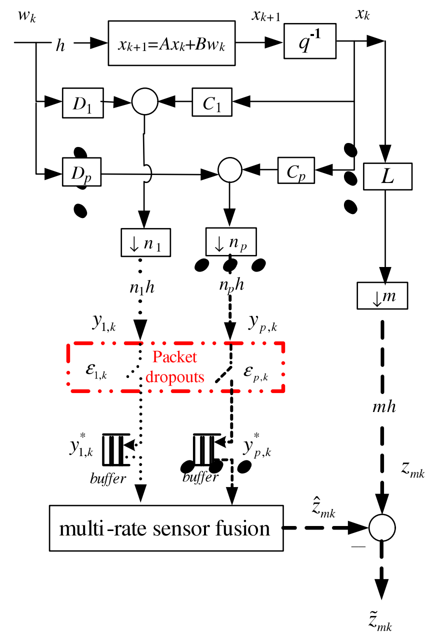

Fig. 1. Multi-rate filtering problem.

2. Problem formulation

As shown in Fig. 1, we present the following linear multi-rate multi-sensor system:

$$x_{k+1} = Ax_k + Bw_k, \tag{1}$$

$$y_{i,k} = C_i x_{n_i k} + D_i w_{n_i k}, \quad (i = 1, 2, \ldots, p), \tag{2}$$

$$z_{mk} = Lx_{mk}, \tag{3}$$

where x is the state vector evolving with a period h; h is a positive real number; y_i is the ith channel sensor measurement vector with a sampling period $n_i h$; z is the vector to be estimated with an updating period mh; both the sampling rate n_i $(i = 1, 2, \ldots, p)$ and the estimation rate m are positive integers; w is a norm-bounded UI vector; the matrices A, B, C_i, D_i and L are known with proper dimensions. In (1)–(3), there are $p+2$ periods: the state updating period h, the p sensor sampling periods $n_1 h, \ldots, n_p h$, and the estimation output period mh.

Moreover, the sensors and a fusion unit are considered to be spatially separated and connected via network media pulsed buffers. Each buffer records only the latest received measurement in the corresponding channel. Due to PDs, the buffered measurement could be different from the original measurement as follows:

$$y^*_{i,k} = \epsilon_{i,k} y_{i,k} + (1 - \epsilon_{i,k}) y^*_{i,k-1} \tag{4}$$

where the stochastic parameter $\epsilon_{i,k}$ takes the value of 0 or 1, representing PD or not. Similar to Sahebsara et al. (2007) and Sun et al. (2008c), $\epsilon_{i,k}$ is unknown in the estimator design and satisfies the following assumption.

Assumption 1. The stochastic parameters $\epsilon_{i,k}$'s are Bernoulli distributed white sequences satisfying

$$\textbf{prob}\left\{\epsilon_{i,k} = 1\right\} = \alpha_i, \quad 0 < \alpha_i \le 1, \tag{5}$$

where α_i is the probability of successful packet transmission for the ith channel sensor measurement.

The main requirements of multi-rate estimator design for (1)–(5) under Assumption 1 in the fusion unit are:

- the estimator parameters can be calculated off-line, which is desirable in real-time applications;