Automatica 46 (2010) 452–459

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Speed regulation in steering-based source seeking[☆]

Nima Ghods, Miroslav Krstic*

Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA

ARTICLE INFO

Article history:
Received 14 July 2009
Received in revised form
23 October 2009
Accepted 16 November 2009
Available online 5 December 2009

Keywords:
Extremum seeking
Adaptive control
Nonholonomic unicycle
Autonomous agents
Averaging

ABSTRACT

The simplest strategy for extremum seeking-based source localization, for sources with unknown spatial distributions and nonholonomic unicycle vehicles without position measurement, employs a constant positive forward speed. Steering of the vehicle in the plane is performed using only the variation of the angular velocity. While keeping the forward speed constant is a reasonable strategy motivated by implementation with aerial vehicles, it leads to complexities in the asymptotic behavior of the vehicle, since the vehicle cannot settle—at best it can converge to a small-size attractor around the source. In this paper we regulate the forward velocity, with the intent of bringing the vehicle to a stop, or as close to a stop as possible. The vehicle speed is controlled using simple derivative-like feedback of the sensor measurement (the derivative is approximated with a washout filter) to which a speed bias parameter V_c is added. The angular velocity is tuned using standard extremum seeking. We prove two results. For V_c in a certain range around zero, we show that the vehicle converges to a ring around the source and on average the limit of the vehicle's heading is either directly away or towards the source. For other values of $V_c > 0$, the vehicle converges to a ring around the source and it revolves around the source. Interestingly, the average heading of this revolution around the source is more outward than inward—this is possible because the vehicle's speed is not constant, it is lower during the outward steering intervals and higher during the inward steering intervals. The theoretical results are illustrated with simulations.

1. Introduction

Motivation. In the rapidly growing literature on coordinated motion control and autonomous agents, "autonomous" never means deprivation of position information. The vehicles are always assumed to have global positioning system (GPS) and/or inertial navigation system (INS) on board. There is, however, interest in developing vehicles with greater autonomy, free of position measurements. The reasons are two-fold: (1) applications underwater, under ice, or in buildings and "urban canyons" where GPS is unavailable, and (2) the high cost of INS systems that remain accurate over extended periods of time.

In previous papers, (Cochran & Krstic, 2007; Zhang, Arnold, Ghods, Siranosian, & Krstic, 2007), we considered the problem of seeking the source of a scalar signal using a nonholonomic vehicle with no position information. We designed two distinct strategies—keeping the angular velocity constant and tuning the forward speed by extremum seeking (Zhang et al., 2007); and keeping the forward speed constant and tuning the angular velocity by extremum seeking (Cochran & Krstic, 2007). The strategy in Zhang et al. (2007) generates vehicle motions that resemble triangles, rhombi, or stars (with arc-shaped sides), which drift towards the source, resulting in periodic motions around the source. The strategy in Cochran and Krstic (2007) generates motions that sinusoidally converge towards the source and settle into an *almost periodic* (in a mathematical sense of the term) motion in a ring around the source. While the proof of the result (Cochran & Krstic, 2007) is more challenging, the vehicle motion is much more efficient than with the strategy in Zhang et al. (2007), since the simple tuning of the heading results in trajectories where the distance of the vehicle from the source decreases monotonically.

Contribution. Neither of the strategies in Cochran and Krstic (2007) and Zhang et al. (2007) are ideal, since Zhang et al. (2007) sacrifice the transients, whereas Cochran and Krstic (2007) complexify the asymptotic performance. In this paper we aim for the best of both worlds, but not by simply combining the strategies in Cochran and Krstic (2007) and Zhang et al. (2007). We propose something more elegant, a strategy that partly simplifies the approach in Cochran and Krstic (2007), while adding a simple derivative-like feedback to a nominal forward speed V_c. This feedback allows the vehicle to slow down as it gets closer to the source and converge closer to the source without giving up convergence speed.

[☆] This work was supported by Los Alamos National Laboratory and Office of Naval Research (ONR). This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Warren E. Dixon under the direction of Editor Andrew R. Teel.

* Corresponding author. Tel.: +1 858 822 1374; fax: +1 858 822 3107.
E-mail addresses: nghods@ucsd.edu (N. Ghods), krstic@ucsd.edu (M. Krstic).

0005-1098/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2009.11.023

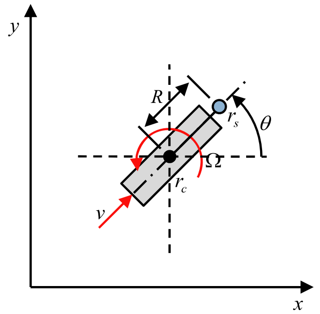

Fig. 1. The notation used in the model of vehicle sensor and center dynamics.

We prove two results, for quadratic signal fields that decay with the distance from the source. For V_c in a certain range around zero, we show that the vehicle converges to a ring around the source and on average the limit of the vehicle's heading is either directly away or towards the source. For other values of $V_c > 0$, the vehicle converges to a ring around the source and it revolves around the source. Interestingly, the average heading of this revolution around the source is more outward than inward—this is possible because the vehicle's speed is not constant, it is lower during the outward steering intervals and higher during the inward steering intervals. The theoretical results are illustrated with simulations. A simulation is also done to consider that case of Rosenbrock function as the signal field.

Source Seeking and Prior Literature. Multi-agent and GPS-enabled source seeking problems have been solved in Ogren, Fiorelli, and Leonard (2004) and Porat and Neohorai (1996). Realistic source seeking formulations require incorporation of nonholonomic constraints on the vehicle models. Such constraints are present in the standard unicycle model, which has been the basis of numerous studies in vehicle formation control, including Justh and Krishnaprasad (2004), Klein and Morgansen (2006) and Marshall, Broucke, and Francis (2006). The level set tracing work in Baronov and Baillieul (2008) is also based on the unicycle model. A hybrid strategy for solving the source seeking problem was developed in Mayhew, Sanfelice, and Teel (2008). The key tool in the present work is extremum seeking (Ariyur & Krstic, 2003), which has been advanced or employed in applications by several other authors Adetola and Guay (2007), Biyik and Arcak (2008), Centioli et al. (2005), King et al. (2006), Li, Rotea, Chiu, Mongeau, and Paek (2005), Ou et al. (2007), Peterson and Stefanopoulou (2004), Popovic, Jankovic, Magner, and Teel (2006), Stegath, Sharma, Gregory, and Dixon (2007), Tan, Nesic, and Mareels (2006), Tanelli, Astolfi, and Savaresi (2006), Wang, Yeung, and Krstic (1999), Wang and Krstic (2000) and Zhang, Dawson, Dixon, and Xian (2006).

Organization of the Paper. We start the paper in Section 2 with a description of the vehicle model and extremum seeking scheme. We derive the averaged system in Section 3. We prove local exponential convergence results to ring/annulus-shaped sets around the source in Sections 4 and 5. Section 4 deals with the case of small $|V_c|$, whereas Section 5 deals with medium and large positive values of V_c. Simulation results in Sections 4 and 5 illustrate the distinct behaviors exhibited using different values of V_c. In Section 6 we summarize the set of possible motions and attractors near the source that are achieved for different values of a key design parameter.

2. Vehicle model and control design

We consider a mobile agent modeled as a unicycle with a sensor mounted at a distance R away from the center. The diagram in Fig. 1 depicts the position, heading, angular and forward velocities for the center and sensor. The equations of motion for the vehicle's center are

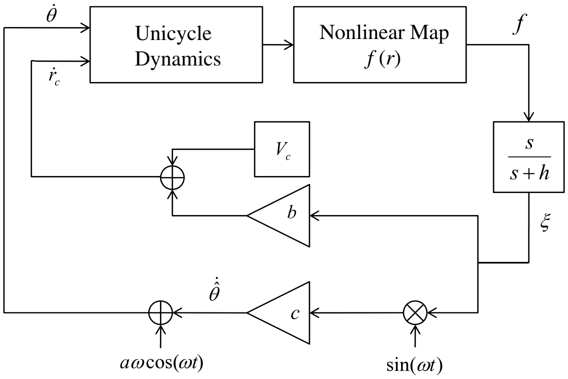

Fig. 2. Block diagram of source seeking via tuning of angular velocity and forward velocity using one reading.

$$\dot r_c = v e^{j\theta} \tag{1}$$

$$\dot\theta = \Omega \tag{2}$$

where r_c is complex variable that represents the center of the vehicle in 2D, θ is the orientation and v and Ω are the forward and angular velocity inputs, respectively. The sensor is located at $r_s = r_c + Re^{j\theta}$. Note that this convenient complex representation of the position would be less useful if extending this work to a 3D setting.

The task of the vehicle is to seek a source that emits a signal (for example, the concentration of a chemical, biological agent, electromagnetic, acoustic, or even thermal signal) which decays as a function of distance away from the source. We assume this signal field is distributed according to an unknown nonlinear map $f(r(x, y))$ which has an isolated local maximum $f^* = f(r^*)$ where r^* is the location of the local maximum. We design a controller that achieves local convergence to r^* without knowledge of the shape of f, using only the measurement $f(r_s)$.

We employ extremum seeking to tune the angular velocity (Ω) directly and the forward velocity (v) indirectly. This scheme is depicted by the block diagram in Fig. 2. The control laws are given by

$$\Omega = a\omega\cos(\omega t) + c\xi\sin(\omega t) \tag{3}$$

$$v = V_c + b\xi, \tag{4}$$

where ξ is the output of the washout filter, namely, of the approximate differentiator of $f(r_s, t)$. The performance can be influenced by the parameters a, c, b, R, h, ω and V_c. We tune angular velocity Ω with the basic extremum seeking tuning law, which has a perturbation term, $a\omega\cos(\omega t)$, to excite the system. The $\xi\sin(\omega t)$ term estimates the angular gradient of the map.

The forward velocity $v = V_c + b\xi$ is chosen using the following intuition. When the vehicle is approaching the source, heading straight towards it, the sensor reading is increasing and hence $\xi > 0$. It is reasonable to speed up the vehicle when it is going towards the source. Conversely, when the vehicle is past the source and the signal reading is decreasing, i.e., $\xi < 0$, the vehicle should be slowed down, which (4) achieves.

We stress that the steering feedback (3) does not employ the nonlinear damping introduced in Cochran and Krstic (2007). The damping needed to exponentially stabilize the average equilibria is provided by the forward speed feedback (4).

3. The average system

We focus on maps which depend on the distance from the source only. Since our goal is only the establishment of local