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Robust adaptive control of a class of nonlinear strict-feedback discrete-time systems with exact output tracking[☆]

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ABSTRACT

In this paper, adaptive control is studied for a class of single-input–single-output (SISO) nonlinear discrete-time systems in strict-feedback form with nonparametric nonlinear uncertainties of the Lipschitz type. To eliminate the effect of the nonparametric uncertainties in an unmatched manner, a novel future states prediction is designed using states information at previous steps to compensate for the effect of uncertainties at the current step. Utilizing the predicted future states, constructive adaptive control is developed to compensate for the effects of both parametric and nonparametric uncertainties such that global stability and asymptotical output tracking is achieved. The effectiveness of the proposed control law is demonstrated in the simulation.

1. Introduction

Robustness in adaptive control has been the subject of much research in both continuous-time and discrete-time, because modeling uncertainties may result in poor performance and even instability of the closed-loop system as observed by Egardt (1979) and Tao (2003). To enhance the robustness of the adaptive control system, many update law modifications were proposed, such as normalization (Goodwin & Mayne, 1987; Tao, 2003) where a normalization term is employed; deadzone method (Egardt, 1979; Peterson & Narendra, 1982) which stops the adaptation when the error signal is smaller than a threshold; projection method (Khalil, 1996; Zhang, Wen, & Soh, 1999, 2001) which projects the parameter estimates into a limited range; σ-modification (Ioannou & Kokotovic, 1983) which incorporates an additional term; and e-modification (Narendra & Annaswamy, 1989) where the constant σ in the σ-modification is replaced by the absolute value of the output tracking error. These methods make the adaptive closed-loop system robust in the presence of an external disturbance or model uncertainties but sacrifice the tracking performance.

On the other hand, adaptive control using the sliding mode has been extensively studied in continuous-time to deal with modeling uncertainty or external disturbance. Recently, many research results of adaptive sliding mode control have also been reported in the discrete-time (Chen, 2006; Chen, Fukuda, & Young, 2001; Lee & Oh, 1998). In contrast to continuous-time systems for which a sliding mode control can be constructed to eliminate the effect of the general uncertain model nonlinearity, in discrete-time the uncertain nonlinearity is required to be of a small growth rate or globally bounded, but sliding mode control is not able to completely compensate for the effect of nonlinear uncertainties in discrete-time.

As a matter of fact, adaptive control design for discrete-time systems is much more difficult than for continuous-time systems. As indicated in Xie and Guo (2000), when the growth rate of the uncertain nonlinearity is larger than a certain number, even a simple first-order discrete-time system cannot be globally stabilized. In an early work (Lee, 1996) on time-varying systems, it is also pointed out that when the parameter time-variation is large, it may be impossible to construct a global stable control even for a first order system. On the other hand, the main stability analysis tool in discrete-time adaptive control, the Key Technical Lemma in Goodwin and Sin (1984), becomes not applicable for the unknown parameters multiplying nonlinearities that are of growth rates faster than linear.

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Due to these difficulties, discrete-time counterparts of continuous-time systems remain largely unexplored. In most existing robust adaptive control results for systems with nonlinear uncertainties, asymptotical tracking performance cannot be achieved though global stability can be guaranteed. It is interesting and challenging in discrete-time adaptive control to fully compensate for the nonparametric uncertainties and to achieve asymptotic tracking. Some recent successful attempts to completely eliminate a class of nonparametric nonlinear uncertainty were made in Sokolov (2003) and Ma, Lum, and Ge (2007), but the designs were limited in the first order system with unknown scalar parameter. To explore adaptive control with full compensation of the nonlinear uncertainties for a general class of minimum phase linear system, a novel adaptive control design using gradient update law has been developed in Yang, Zhai, Ge, Chai, and Lee (2008).

Recently, nonlinear systems in the lower triangular form have attracted great interest in discrete-time adaptive control area. Adaptive backstepping design in discrete-time has been proposed in Yeh and Kokotovic (1995) for a class of parameter-strict-feedback systems. Later, robust adaptive control has been studied for parameter-strict-feedback systems in Zhang et al. (1999, 2001) using projection in parameter estimates update law. In Zhao and Kanellakopoulos (2002), a novel parameter estimator is proposed for parameter-strict-feedback systems in the absence of any disturbance and model uncertainties and it guarantees the convergence of estimates to the real values in finite steps. However, it is noted in Ge, Yang, and Lee (2008b) that these results on parameter-strict-feedback systems are not directly applicable to more general strict-feedback systems with unknown control gains. Therefore, following the concept of system transformation in Ge, Li, and Lee (2003) and Ge, Yang, and Lee (2008a), future states prediction based adaptive control has been developed in Ge et al. (2008b). The prediction method has also been extended to output prediction in Yang, Ge, and Lee (2009). For a class of strict-feedback systems with partially unknown control gains and nonlinear uncertainty in the control range (matched uncertainty), adaptive control with uncertainty compensation has been studied in Yang, Dai, Ge, and Lee (2009), in which asymptotic tracking is guaranteed.

In this paper, we further study adaptive control of strict-feedback systems with both matched and unmatched uncertainties. Continuous-time adaptive control for this class of systems has been developed in Polycarpou and Ioannou (1996) and Jiang and Praly (1998). However, the nonlinear damping method used in the these works to counteract the nonparametric uncertainties is not applicable to discrete-time systems, even when the nonparametric uncertainties only appear in the control range. One reason is the difference of a quadratic Lyapunov function in discrete-time does not inherit linearity property of differential of counterpart Lyapunov in continuous-time, the other reason is that in the discrete-time system formulation, the current input only affects future states which are not available for feedback at current step. In this paper, future states prediction approach is developed which extends the prediction methods in Ge et al. (2008b) by introducing auxiliary states and their estimates, based on which prediction can be proceeded with compensation for the effect of unmatched uncertain nonlinearities. In addition, a novel deadzone method is proposed to guarantee boundedness of closed-loop signals. By sorting growth orders of closed-loop signals, it is finally proved rigourously that asymptotical tracking is achieved.

Throughout this paper, the following notations are used.

- $\|\cdot\|$ denotes the Euclidean norm of vectors and induced norm of matrices.
- $A := B$ means that B is defined as A.
- $(\,)^{\mathrm{T}}$ represents the transpose of vector.
- $\mathbf{0}_{[p]}$ stands for p-dimension zero vector.
- Z_t^+ represents the set of all integers which are not less than a given integer t.

2. Problem formulation and preliminaries

2.1. System representation

Consider a class of SISO nonlinear discrete-time systems with both parametric and nonparametric uncertainties in the following strict-feedback form:

$$\begin{cases} \xi_i(k+1) = \Theta_i^{\mathrm{T}}\Phi_i(\bar{\xi}_i(k)) + g_i\xi_{i+1}(k) + \upsilon_i(\bar{\xi}_i(k)) \\ \qquad i = 1, 2, \ldots, n-1 \\ \xi_n(k+1) = \Theta_n^{\mathrm{T}}\Phi_n(\bar{\xi}_n(k)) + g_n u(k) + \upsilon_n(\bar{\xi}_n(k)) \\ y(k) = \xi_1(k) \end{cases} \tag{1}$$

where $\bar{\xi}_j(k) = [\xi_1(k), \xi_2(k), \ldots, \xi_j(k)]^{\mathrm{T}}$ are measurable system states, $\forall k \in Z_{-n}^+$, $\Theta_j \in R^{p_j}$, $g_j \in R$, $j = 1, 2, \ldots, n$, are unknown parameters (p_j's are positive integers), $\Phi_j(\bar{\xi}_j(k)) : R^j \to R^{p_j}$ are known vector-valued functions, $\upsilon_i(\bar{\xi}_j(k))$ are nonparametric nonlinear certainties, $k \in Z_{-n}^+$, which can be regarded as nonlinear model uncertainties, $u(k)$ and $y(k)$ are system input and output, respectively. The control objective is to make the output $y(k)$ exactly track a bounded reference trajectory $y_d(k)$ and to guarantee the boundedness of all the closed-loop signals. It is noted that the nonparametric nonlinear uncertainties $\upsilon_i(\cdot)$ are unmatched (out of the control range). Though matched uncertainties (in the control range) have been extensively studied in the robust control literature (Chan, 1994; Chen, 2006; Chen et al., 2001; Myszkorowski, 1994), which guarantee global stability but not asymptotical tracking performance, there are few results on studying compensation of unmatched uncertainties.

Assumption 1. The nonparametric uncertain functions $\upsilon_i(\cdot)$, are Lipschitz functions with Lipschitz coefficients L_{υ_i}, i.e., $|\upsilon_i(\varepsilon_1) - \upsilon_i(\varepsilon_2)| \le L_{\upsilon_i}\|\varepsilon_1 - \varepsilon_2\|$, $\forall \varepsilon_1, \varepsilon_2 \in R^n$, where $\max_{1\le i\le n} L_{\upsilon_i} < \lambda^*$ and λ^* is a small number defined in (49). The system functions, $\Phi_i(\cdot)$, $i = 1, 2, \ldots, n$, are also Lipschitz functions with Lipschitz coefficients L_i.

Assumption 2. The signs of control gains g_i, $(i = 1, 2, \ldots, n)$ are known. Without loss of generality, it is assumed that g_i are positive with known lower bounds $\underline{g}_i > 0$, i.e., $g_i \ge \underline{g}_i > 0$.

Remark 1. As pointed in Xie and Guo (2000), it is impossible to obtain global stability results for discrete-time controlled system when the nonlinear uncertainties are of large growth rates. Thus, it is usual to assume that the nonparametric nonlinear uncertainties are of small growth rates (Chen, 2006; Chen et al., 2001; Myszkorowski, 1994; Zhang et al., 1999, 2001) or even globally bounded (Chen & Narendra, 2001; Tao, 2003) and their growth rates can be guaranteed to be smaller than a specified constant. In the case that the discrete-time model is derived from a continuous-time model, the growth rates of the nonlinear uncertainties can be made small enough by choosing a sufficiently small sampling time T.

Remark 2. The counterpart of system (1) in continuous-time has been studied in Polycarpou and Ioannou (1996) and Jiang and Praly (1998) by combining backstepping design and nonlinear damping method. However, like high gain control, nonlinear damping is not applicable to discrete-time for complete nonlinear uncertainties compensation. In this paper, a novel design is proposed to utilize states information at previous steps to compensate uncertainties at current step.

2.2. Useful definitions and lemmas

Definition 1 (*Chen & Narendra, 2001*). Let $x_1(k)$ and $x_2(k)$ be two discrete-time scalar or vector signals, $\forall k \in Z_t^+$, for any t.