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Brief paper

Robust stability and H_∞-control of uncertain impulsive systems with time-delay[☆]

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ABSTRACT

This paper investigates the problems of robust stability, stabilization and H_∞-control for uncertain impulsive systems with time-delay. The parametric uncertainties are assumed to be time-varying and norm-bounded. Three classes of impulsive systems with time-delay are considered: the systems with stable/stabilizable continuous dynamics and unstable/unstabilizable discrete dynamics, the systems with unstable/unstabilizable continuous dynamics and stable/stabilizable discrete dynamics, and the systems where both the continuous-time dynamics and the discrete-time dynamics are stable/stabilizable. For each class of system, by using the Lyapunov function and Razumikhin-type techniques, sufficient conditions for robust stability, stabilization and H_∞-control are developed in terms of linear matrix inequalities. Numerical examples are given which illustrate the applicability of the theoretical results.

1. Introduction

Impulsive dynamical systems have attracted considerable interest in science and engineering during the past decades because they provide a natural framework for mathematical modeling of many real world evolutionary processes where the states undergo abrupt changes at certain instants. An impulsive dynamical system can be viewed as a hybrid one comprised of three components: a continuous-time differential equation, which governs the motion of the dynamical systems between impulsive or resetting events; a difference equation, which governs the way the system states are instantaneously changed when a resetting event occurs; and a criterion for determining when the states of the systems are to be reset. Stability properties of impulsive dynamical systems have been extensively studied in the literature, we refer to Bainov and Simeonov (1989), Li, Soh, and Xu (1997, 1998), Li, Wen, and Soh (2001), Xu and Chen (2003) and Yang (2001), and the references therein. The impulsive control method based on stability theory of impulsive dynamical systems has been widely used in the stabilization and synchronization of chaotic systems, see, e.g., Wen, Ji, and Li (2007), Yang and Chua (1997), Yang, Yang, and Yang (1997), and Yang (2001). For the impulsive systems with external disturbances, the problems of input-state stability (ISS), dissipativity and H_∞ control have been investigated in Guan, Yao, and Hill (2005), Hespanha, Liberzon, and Teel (2005), Haddad, Chellabonia, and Kablar (2001) and Yao, Guan, Chen, and Ho (2006). However, the corresponding theory for impulsive dynamical systems with time-delay has been relatively less developed. The difficulty in developing such a theory may come from the effects of impulses and time-delay on system behaviors. Most of the results for impulsive dynamical systems with time-delay in the current literature focus on the stability analysis (Anokhin, Berezansky, & Braverman, 1995; Liu & Ballinger, 2001; Wang & Liu, 2005; Yang & Xu, 2007). There have been very few results reported on controller synthesis for impulsive control systems with time-delay.

This paper will study the problems of robust exponential stability, robust stabilization and robust H_∞ control for uncertain impulsive systems with time-delay. Different from the work on H_∞ control of impulsive systems without delay in Guan et al. (2005) and Yao et al. (2006), where both the continuous dynamics and discrete dynamics are required to be stable/stabilizable, we will consider more general types of impulsive delayed systems. That is, we will divide the impulsive control systems with time-delay into three classes: the systems with stable/stabilizable continuous dynamics and unstable/unstabilizable discrete dynamics, the systems with unstable/unstabilizable continuous dynamics and stable/stabilizable discrete dynamics, and the systems where both the continuous-time dynamics and the discrete-time dynamics are stable/stabilizable. The first class of impulsive systems corresponds

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to the case when the continuous dynamics are subjected to impulsive perturbations, while the second class of impulsive systems corresponds to the case when impulses are employed to stabilize the unstable continuous dynamics. It is noted that the standard methods from H_∞-control for continuous delayed systems are not suitable for the H_∞ control for the first and the second classes of impulsive delayed systems. Thus the stability and control issues of these two classes of impulsive systems with time-delay are of theoretical and practical importance. In this paper, we will first establish the delay-independent exponential stability and state-feedback stabilization criteria in terms of linear matrix inequalities (LMIs) using Lyapunov–Razumikhin techniques. Then based on the stabilization results, some new analysis techniques will be developed to derive the sufficient conditions for H_∞-control of the above three classes of impulsive delayed systems. Finally, four examples will be given to demonstrate the effectiveness of the proposed approach.

2. Problem statement

In the sequel, if not explicitly given, matrices are assumed to have compatible dimensions. The notation $M > (\geq, <, \leq)0$ is used to denote a symmetric positive-definite (positive-semidefinite, negative, negative-semidefinite) matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ represent the minimum and maximum eigenvalues of the corresponding matrix, respectively. $\|\cdot\|$ denotes Euclidean norm for vectors or the spectral norm of matrices. $\mathbb{N}$ denotes the set of positive integers. For $\tau > 0$, let $PC([-\tau, 0], \mathbb{R}^n)$ denote the set of piecewise right continuous function $\phi : [-\tau, 0] \to \mathbb{R}^n$ with the norm defined by $\|\phi\|_\tau = \sup_{-\tau\leq\theta\leq 0}\|\phi(\theta)\|$.

Consider uncertain linear impulsive systems with time-delay described by the following state equation

$$\left.\begin{array}{l}\dot{x}(t) = A(t)x(t) + A_1(t)x(t-\tau(t)) + f(t, x_t)\\ \quad + B_1(t)u_c(t) + H_1 w(t), \quad t \neq t_k,\\ \Delta x(t) = (C(t) - I)x(t^-) + B_I(t)u_d(t^-), \quad t = t_k,\\ z(t) = E(t)x(t) + B_2(t)u(t) + H_2 w(t),\\ x(t_0+\theta) = \phi(\theta), \quad t_0 = 0, \theta \in [-\tau, 0],\end{array}\right\} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u_c(t) \in \mathbb{R}^{m_1}$ is the continuous control input, $u_d(t) \in \mathbb{R}^{m_2}$ is the impulsive control input, $w(t) \in \mathbb{R}^p$ is the disturbance input which belongs to $L_2[0, \infty)$, and $z(t) \in \mathbb{R}^q$ is the controlled output. $\tau(t)$ is a time-varying delay satisfying $0 \leq \tau(t) \leq \tau$. $f(t, x_t)$ is the unknown nonlinear perturbation. We assume that $f : \mathbb{R}^+ \times PC([-\tau, 0], \mathbb{R}^n) \to \mathbb{R}^n$ is continuous and satisfies the following condition:

$$\|f(t, x_t)\|^2 \leq \|Gx(t)\|^2 + \|G_1 x(t-\tau(t))\|^2, \quad t \geq 0. \tag{2}$$

where G and G_1 are known constant matrices. $\phi \in PC([-\tau, 0], \mathbb{R}^n)$ is the initial condition of the state. $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ describes the state jumping at impulsive time instant $t = t_k$, where $x(t_k^+) = x(t_k) = \lim_{h\to 0+} x(t_k + h)$, $x(t_k^-) = \lim_{h\to 0-} x(t_k + h)$, $k = 1, 2, \ldots,$ and $0 < t_1 < t_2 < \cdots < t_k < \cdots$ ($t_k \to \infty$ as $k \to \infty$). H_1, H_2 are known constant matrices with appropriate dimensions. $A(t), B_1(t), E(t), B_2(t), C(t), B_I(t)$ are matrix functions with time-varying uncertainties, that is, $A(t) = A + \Delta A(t)$, $A_1(t) = A_1 + \Delta A_1(t)$, $B_1(t) = B_1 + \Delta B_1(t)$, $E(t) = E + \Delta E(t)$, $B_2(t) = B_2 + \Delta B_2(t)$, $C(t) = C + \Delta C(t)$, $B_I(t) = B_I + \Delta B_I(t)$, where $A, A_1, B_1, E, B_2, C, B_I$ are known real constant matrices, $\Delta A(t), \Delta A_1(t), \Delta B_1(t), \Delta E(t), \Delta B_2(t), \Delta C(t)$, and $\Delta B_I(t)$ are unknown matrices representing time-varying parameter uncertainties. We assume that the uncertainties are norm-bounded and can be described as

$$\left.\begin{array}{l}[\Delta A(t)\ \Delta A_1(t)\ \Delta B_1(t)] = D_1 F_1(t)\,[N_1\ \ N_d\ N_{b1}],\\ [\Delta E(t)\ \Delta B_2(t)] = D_2 F_2(t)\,[N_2\ N_{b2}],\\ [\Delta C(t)\ \Delta B_I(t)] = D_0 F_3(t)\,[N_0\ \ N_I],\end{array}\right\} \tag{3}$$

where $D_0, D_1, D_2, N_0, N_1, N_2, N_{b1}, N_{b2}, N_d, N_I$ are known real constant matrices and $F_i(t)$ are unknown matrix functions satisfying $\|F_i(t)\| \leq 1$, $i = 1, 2, 3$. It is assumed that the elements of $F_i(t)$ are Lebesgue measurable, $i = 1, 2, 3$.

In what follows, we will divide three cases to establish the robust stability, robust stabilization and robust H_∞ control of system (1) by using Lyapunov–Razumikhin techniques. We denote by $\mathcal{S}_{\min}(\beta)$ the class of impulse time sequences that satisfy $\inf_k\{t_k - t_{k-1}\} \geq \beta$, and denote by $\mathcal{S}_{\max}(\beta)$ the class of impulse time sequences that satisfy $\sup_k\{t_k - t_{k-1}\} \leq \beta$.

3. Robust stability and robust stabilization

This section considers the exponential stability and stabilization of system (1) and its robustness. For this purpose, we restrict our study to the case of $w(t) = 0$, i.e.,

$$\left.\begin{array}{l}\dot{x}(t) = A(t)x(t) + A_1(t)x(t-\tau(t)) + f(t, x_t)\\ \quad + B_1(t)u_c(t), \quad t \neq t_k,\\ \Delta x(t) = (C(t) - I)x(t^-) + B_I(t)u_d(t^-), \quad t = t_k,\\ x(t_0+\theta) = \phi(\theta), \quad t_0 = 0, \theta \in [-\tau, 0].\end{array}\right\} \tag{4}$$

First, we introduce the definition of exponential stability.

Definition 1. For a given class $\mathcal{S}$ of admissible impulse time sequences, system (4) with $u_c(\cdot) = 0$ and $u_d(\cdot) = 0$ is said to be robustly exponentially stable over $\mathcal{S}$ if there exists a scalar $\nu > 0$ such that, for every $\varepsilon > 0$, there is a scalar $\delta > 0$ such that $\phi \in PC([-\tau, 0], \mathbb{R}^n)$ with $\|\phi\|_\tau \leq \delta$ implies $\|x(t, t_0, \phi)\| < \varepsilon \exp(-\nu(t - t_0))$, $t \geq t_0$, for all admissible uncertainties satisfying (3) and for every impulse time sequence in $\mathcal{S}$.

Next, we develop Lyapunov–Razumikhin techniques to establish the sufficient conditions for exponential stability of system (4) with $u_c(\cdot) = 0$ and $u_d(\cdot) = 0$.

Theorem 1. *Consider system (4) with $u_c(\cdot) = 0$ and $u_d(\cdot) = 0$. Assume that for a prescribed scalar $\beta > 0$, there exist a matrix $X > 0$ and positive scalars μ, α, d, ε_1, and ε_2 such that the following matrix inequalities hold:*

$$\begin{bmatrix}\Psi & A_1X & dI & XN_1^{\mathrm{T}} & XG^{\mathrm{T}} & 0\\ * & -\alpha X & 0 & XN_d^{\mathrm{T}} & 0 & XG_1^{\mathrm{T}}\\ * & 0 & -dI & 0 & 0 & 0\\ * & * & 0 & -\varepsilon_1 I & 0 & 0\\ * & 0 & 0 & 0 & -dI & 0\\ 0 & * & 0 & 0 & 0 & -dI\end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix}-\mu X & XC^{\mathrm{T}} & XN_0^{\mathrm{T}}\\ * & -X + \varepsilon_2 D_0 D_0^{\mathrm{T}} & 0\\ * & 0 & -\varepsilon_2 I\end{bmatrix} \leq 0, \tag{6}$$

where $\Psi = AX + XA^{\mathrm{T}} + (\alpha g(\mu) + \ln\mu/\beta)X + \varepsilon_1 D_1 D_1^{\mathrm{T}}$, $g(\mu) = \mu$ if $\mu \geq 1$ and $g(\mu) = 1/\mu$ if $\mu < 1$. Then for any bounded time-delay $\tau(t)$, when $\mu > 1$, system (4) with $u_c(\cdot) = 0$ and $u_d(\cdot) = 0$ is robustly exponentially stable over $\mathcal{S}_{\min}(\beta)$; when $\mu = 1$, system (4) with $u_c(\cdot) = 0$ and $u_d(\cdot) = 0$ is robustly exponentially stable for any impulsive time sequence $\{t_k\}$; when $\mu < 1$, system (4) with $u_c(\cdot) = 0$ and $u_d(\cdot) = 0$ is robustly exponentially stable over $\mathcal{S}_{\max}(\beta)$.

Proof. Set $P = X^{-1}$, $\lambda_0 = \lambda_{\min}(P)$ and $\lambda_1 = \lambda_{\max}(P)$. For any $\phi \in PC([-\tau, 0], \mathbb{R}^n)$, we denote $x(t, t_0, \phi)$ by $x(t)$ and set $V(t) = x^{\mathrm{T}}(t)Px(t)$. Pre- and post-multiplying (6) by diag$\{P, P, I\}$ and combining with Schur complements yields

$$-\mu P + C^{\mathrm{T}}(P^{-1} - \varepsilon_2 D_0 D_0^{\mathrm{T}})^{-1}C + \varepsilon_2^{-1}N_0^{\mathrm{T}}N_0 \leq 0. \tag{7}$$

Then using the second equation of (4), and applying (b) of Lemma 2.2 in Wang, Xie, and de Souza (1992) and (7), one has