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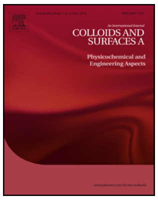

Cracks and fingers: Dynamics of ductile fracture in an aqueous foam

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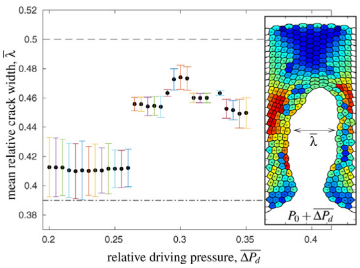
HIGHLIGHTS

- A discrete network model for ductile fracture of a foam is analysed.
- The morphology of the crack tip is estimated for a range of driving pressures.
- The fracture pattern resembles Saffman-Taylor fingering in Non-Newtonian fluids.

GRAPHICAL ABSTRACT



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ABSTRACT

Fracture of a quasi-two-dimensional aqueous foam by injection of air can occur via two distinct mechanisms, termed brittle and ductile, which are analogous to crack modes observed for crystalline atomic solids such as metals. In the present work we focus on the dynamics and morphology of the ductile process, in which no films between bubbles are broken. A network modeling approach allows detailed analysis of the foam morphology from individual bubbles to the shape of the propagating crack. This crack develops similarly to fingering instabilities in Hele–Shaw cells filled with homogeneous fluids. We show that the observed width and shape of the crack are compatible this interpretation, and that the discreteness of the bubble structure provides symmetry perturbations and limiting scales characteristic of anomalous fingering. The model thus bridges the gap between fracture of the solid foam lattice and instability growth of interfaces in a fluid system.

1. Introduction

The analogue between the structure of a gas–liquid foam and the structure of a crystalline atomic solid (like metal) has long been established [1]. Apart from geometric analogies of defect motion and generation, as well as plasticity on the atomic scale [2,3], recent research efforts have investigated how a qualitative understanding of fracture processes acting in a foam can illuminate the fracture morphology and dynamics in an atomic solid. In addition to such fundamental questions, the study of foam fracture is also of great relevance in applications relying on the integrity of liquid foams, e.g. in enhanced oil recovery [4], the manufacture of porous metallic solids [5], or in mineral flotation [6].

A prototypical system for the study of foam fracture has been developed both experimentally and in subsequent modeling efforts using a quasi-two-dimensional foam, *i.e.*, a single layer of bubbles between the parallel plates of a Hele–Shaw cell. The experimental realization of this concept has considered the phenomena upon injection of air into such a foam at a given pressure difference from one end to the other (open) end of the foam, while the bubbles are laterally confined to a rectangular region (see Fig. 1). Fracture is then observed to proceed along the direction of pressure drop in one of two distinct mechanisms depending on the rate of applied pressure. Much as in crystalline atomic solids, these mechanisms can be termed brittle and ductile: in the former case observed for

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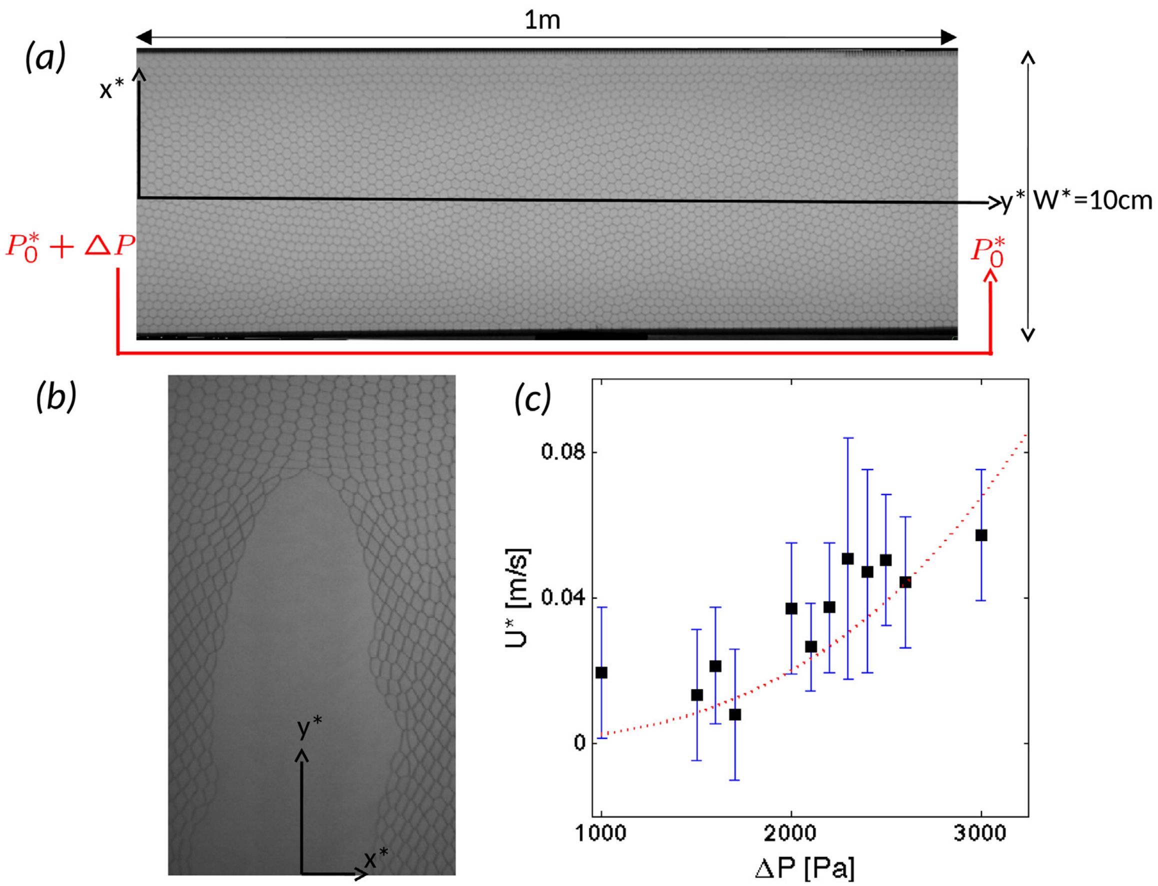

Fig. 1. (a) Top view of a Hele–Shaw cell of width W^* = 10 cm and length > 1 m filled with a single layer of foam bubbles (edge length $L \approx 2$ mm). A pressure difference is applied along the channel (y-direction) as indicated. (b) Typical experimental image of a propagating ductile crack (finger). The experimental field of view is limited to approximately the displayed length of the crack. The relative crack width determined from this image is at least $\bar{\lambda} \approx 0.39$. (c) Experimental data of ductile crack speed U^* vs. applied driving pressure ΔP (modified from [8]). The dashed line indicates the power law $U^* \propto \Delta P^{3/2}$, consistent with the modeling assumptions of the present work (cf. Eq. (3b)).

large rates of applied pressure, very fast cleavages of the foam are sustained with very little deformation around the crack surface. These brittle cracks proceed by successive breakage of the thin films between foams in almost perfect sequence along the pressure gradient [7,8]. In the latter case, observed for smaller rates of applied pressure, a much slower air/foam interface propagation is observed [9], morphologically resembling a fingering instability in homogeneous fluid systems described by Saffman and Taylor [10] and studied extensively as an example of nonlinear instability and shape selection [11–13]. While the system can also exhibit a brittle-to-ductile transition [8] between these modes, we will here focus on the ductile case only, see Fig. 1. A qualitatively similar ductile foam invasion mechanism has also been observed in foams with a large bubble in the interior [14,15], while other fingering-type instabilities have been observed in foams where an interior bubble is continuously inflated [16,17]. Experiments indicate that yield stress can significantly influence the fingering pattern in flowing polymer gels, but the effect of yield stress is notably absent in foam fingering [18].

We have previously developed a large-scale network model to understand and quantify both modes of foam fracture under an applied driving pressure [19], based on a methodology developed for studying coalescence in molten metallic foams [20,21]. This approach uses a simplified description of each of the fluid structures and incorporates an explicit film rupture criterion in the brittle case [22]. In the present study of ductile effects, the fast time scales of film rupture are unimportant, and the model focus lies on the (non-Newtonian) resistance of the foam bubbles when driven through the gap between the plates.

In this paper we utilize our network model to understand and quantify ductile fracture in large initially regular arrays, and connect our results to continuum theory of fingering in fluids. The model as it pertains to ductile fracture is briefly described in Section 2. It is then used to predict features of crack morphology and dynamics in Section 3, such as the mean crack width and crack tip speed as a function of the applied driving pressure. Finally, in Section 4 these numerical predictions are interpreted and compared to results on fingering in continuum fluid systems, including anomalous fingering in Newtonian fluids and shear thinning fluids. Section 5 provides conclusions and a discussion on the interrelation between fracture and fingering.

2. Ductile fracture model

We previously constructed a network model for foam fracture encompassing brittle and ductile effects [19] in a long Hele–Shaw cell of thickness (gap between the plates) b^* and lateral width W^*. In this study we use the same equations and notation with only small modifications which are detailed below. The domain is spanned by Cartesian coordinates, $\mathbf{x}^* = (x^*, y^*, z^*)$, where x^* is oriented across the channel width, y^* is in the direction of driving and z^* spans the Hele–Shaw cell thickness.

Throughout this study we focus attention on the ductile fracture mode and ignore the fast processes associated with film rupture (see [22] for details on film breakage in this setting). We denote ρ^* and μ^* as the density and dynamic viscosity of the water phase, respectively, and γ^* as surface-tension coefficient of the air-water interface (suitably modified by the presence of surfactants). We

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