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Brief paper

Output feedback adaptive control of a class of nonlinear discrete-time systems with unknown control directions☆

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ABSTRACT

In this paper, output feedback adaptive control is investigated for a class of nonlinear systems in output-feedback form with unknown control gains. To construct output feedback control, the system is transformed into the form of the NARMA (nonlinear-auto-regressive-moving-average) model, based on which future output prediction is carried out. With employment of the predicted future output, a constructive output feedback adaptive control is given with the discrete Nussbaum gain exploited to overcome the difficulty due to unknown control directions. Under the global Lipschitz condition of the system functions, the boundedness of all the closed-loop signals and asymptotical output tracking are achieved by the proposed control. Simulation results are presented to show the effectiveness of the proposed approach.

1. Introduction

In the last two decades, the problem of controlling nonlinear systems with unknown control directions has received a great deal of attention for the continuous-time systems (Ge & Wang, 2003; Ge, Hong, & Lee, 2004; Kaloust & Qu, 1995; Lozano, Collado, & Mondie, 1990; Nussbaum, 1983; Ryan, 1994; Ye & Jiang, 1998). The control directions, defined as signs of the control gains, are normally required to be known *a priori* in adaptive control literature. When the signs of control gains are unknown, the adaptive control problem becomes much more difficult, since we cannot decide the direction along which the control operates. The unknown control directions problem had remained open till the Nussbaum-type gain was first introduced in Nussbaum (1983) for adaptive control of first order continuous-time systems. Later, the Nussbaum gain was adopted in the adaptive control of linear systems with nonlinear uncertainties (Ryan, 1994) to counteract the lack of *a prior* knowledge of control directions. Towards high order nonlinear systems, backstepping with Nussbaum function was then developed for general nonlinear systems in the triangular structure, with constant control gains (Ye & Jiang, 1998), and time varying control gains (Ge & Wang, 2003). Recently, the results have also been extended to nonlinear systems with general unknown nonlinear functions by using neural network parametrization techniques (Ge et al., 2004). It should be mentioned that besides Nussbaum gain, some other methods to deal with unknown control directions have also been developed in the literature (Kaloust & Qu, 1995; Lozano et al., 1990), but the application of these methods is restricted to certain systems and is not as general as Nussbaum gain.

In contrast to the aforementioned results for continuous-time systems, their discrete-time counterparts remain largely unexplored. In addition, many continuous-time control methods may be not suitable for discrete-time systems, e.g., the backstepping design proposed in Krstic, Kanellakopoulos, and Kokotovic (1995), a crucial ingredient for the development of solutions to many continuous-time adaptive nonlinear problems, may be not directly applicable to discrete-time systems (Ge, Li, & Lee, 2003). To develop a discrete-time counterpart of continuous-time adaptive backstepping, the approach that "looks ahead" and chooses the control law to force the states to acquire their desired values was proposed in Yeh and Kokotovic (1995). But the proposed adaptive control is not applicable to systems with unknown control gains. On the other hand, the discrete Nussbaum gain, a counterpart of the continuous-time Nussbaum gain, was less exploited since it was proposed in Lee and Narendra (1986) for adaptive control discrete-time linear systems. In this paper, we will employ discrete Nussbaum gain for adaptive output feedback control of a class of discrete-time nonlinear systems with finite unknown control gains. Under the global Lipschitz condition of the system functions, the proposed adaptive control guarantees asymptotical tracking performance.

Throughout this paper, the following notations are used in order.

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- $\|\cdot\|$ denotes the Euclidean norm of vectors and induced norm of matrices.
- $[\,]^{\mathrm{T}}$ represents the transpose of a vector or a matrix.
- $\mathbf{0}_{[p]}$ stands for a p-dimension zero vector.
- $\hat{()}$ and $\tilde{()}$ denote the estimate of parameters and estimation error, respectively.

2. Problem formulation and preliminaries

2.1. System representation

The systems we consider in this paper are in the output-feedback form with unknown control gains as follows:

$$\begin{cases} x_1(k+1) = \Theta_1^{\mathrm{T}}\Phi_1(x_1(k)) + g_1x_2(k) \\ x_2(k+1) = \Theta_2^{\mathrm{T}}\Phi_2(x_1(k)) + g_2x_3(k) \\ \vdots \\ x_n(k+1) = \Theta_n^{\mathrm{T}}\Phi_n(x_1(k)) + g_nu(k) \\ y(k) = x_1(k) \end{cases} \tag{1}$$

where $\Theta_i \in R^{p_i}$ are the vectors of unknown constant parameters and $g_i \in R$ are unknown control gains, $\Phi_i(\cdot) : R \to R^{p_i}$, $i = 1, 2, \ldots, n$, are known nonlinear vector functions, $x_1(k), x_2(k), \ldots, x_n(k)$ are the system states, $n \geq 1$ is system order. It is noted that the nonlinearities that are multiplied by the unknown vector parameters depend only on the output $y(k) = x_1(k)$, which is the only measured state. This justifies the name of “output-feedback” form.

Assumption 1. The system functions $\Phi_i(\cdot)$ are Lipschitz functions, i.e., $\|\Phi_i(\varepsilon_1) - \Phi_i(\varepsilon_2)\| \leq L_i\|\varepsilon_1 - \varepsilon_2\|, \forall\varepsilon_1, \varepsilon_2 \in R$, $1 \leq i \leq n$, with finite constants L_i. The control gains $g_i \neq 0$.

It should be noted that neither the sign of g_i (the control direction) nor the upper or lower bound of g_i are assumed to be known in the paper. If the control gains g_i's are all ones, the system becomes in the so called “parametric-output-feedback” form studied in Zhao and Kanellakopoulos (2002) where the authors proposed a parameter estimator that guarantees the estimates converge to the true values in finite steps. The control objective in this paper is to design an output feedback control $u(k)$ such that the output $y(k)$ tracks a bounded reference trajectory $y_d(k)$ and all the closed-loop signals are guaranteed to be bounded.

2.2. Preliminaries

Definition 1 (*Chen & Narendra, 2001*). Let $x_1(k)$ and $x_2(k)$ be two discrete-time scalar or vector signals

- We denote $x_1(k) = O[x_2(k)]$, if there exist positive constants m_1, m_2 and k_0 such that $\|x_1(k)\| \leq m_1 \max_{k'\leq k}\|x_2(k')\| + m_2, \forall k > k_0$.
- We denote $x_1(k) = o[x_2(k)]$, if there exists a discrete-time function $\alpha(k)$ satisfying $\lim_{k\to\infty}\alpha(k) \to 0$ and a constant k_0 such that $\|x_1(k)\| \leq \alpha(k)\max_{k'\leq k}\|x_2(k')\|, \forall k > k_0$.
- We denote $x_1(k) \sim x_2(k)$ if they satisfy $x_1(k) = O[x_2(k)]$ and $x_2(k) = O[x_1(k)]$.

Lemma 1 (*Ge, Yang, and Lee, 2008a*). *Under Assumption 1, for $i = 1, 2, \ldots, n$, the states and input of system (1) satisfy*

$$\xi_i(k) = O[y(k+i-1)], \qquad u(k) = O[y(k+n)].$$

Definition 2 (*Yang, Ge, Xiang, Chai, and Lee, 2008*). Consider a discrete nonlinear function $N(x(k))$ defined on a sequence $x(k)$ with $x_s(k) = \sup_{k'\leq k}\{x(k')\}$. $N(x(k))$ is a discrete Nussbaum gain if and only if it satisfies the following two properties:

(i) If $x_s(k)$ increases without bound, then for any given constant δ_0

$$\sup_{x_s(k)\geq\delta_0}\frac{S_N(x(k))}{x_s(k)} = +\infty, \qquad \inf_{x_s(k)\geq\delta_0}\frac{S_N(x(k))}{x_s(k)} = -\infty.$$

(ii) If $x_s(k) \leq \delta_1$, then $|S_N(x(k))| \leq \delta_2$ with some positive constants δ_1 and δ_2.

where $S_N(x(k))$ is defined with $\Delta x(k) = x(k+1) - x(k)$ as follows:

$$S_N(x(k)) = \sum_{k'=0}^{k} N(x(k'))\Delta x(k'). \tag{2}$$

In this paper, for adaptive control of system (1), the discrete Nussbaum gain $N(x(k))$ proposed in Lee and Narendra (1986) will be exploited, which requires the sequence $x(k)$ to satisfy

$$x(k) \geq 0, \quad \forall k, |\Delta x(k)| = |x(k+1) - x(k)| \leq \delta_0. \tag{3}$$

Lemma 2 (*Ge, Yang, and Lee, 2008b*). *Let $V(k)$ be a positive definite function defined $\forall k$, $N(x(k))$ be a discrete Nussbaum gain, and θ be a nonzero constant. If the following inequality holds, $\forall k$*

$$V(k) \leq \sum_{k'=k_1}^{k}(c_1 + \theta N(x(k')))\Delta x(k') + c_2x(k) + c_3 \tag{4}$$

where c_1, c_2 and c_3 are some constants, k_1 is a positive integer, then $V(k)$, $x(k)$ and $\sum_{k'=k_1}^{k}(c_1 + \theta N(x(k')))\Delta x(k') + c_2x(k) + c_3$ must be bounded, $\forall k$.

3. System transformation

To facilitate the control design, let us consider a state transformation such that $\xi_i(k) = x_i(k)\prod_{j=0}^{i-1}g_j$ with $g_0 = 1$, which transforms system (1) into the following form:

$$\begin{cases} \xi_1(k+1) = \Theta_f^{\mathrm{T}}\Phi_{f1}(\xi_1(k)) + \xi_2(k) \\ \xi_2(k+1) = \Theta_f^{\mathrm{T}}\Phi_{f2}(\xi_1(k)) + \xi_3(k) \\ \vdots \\ \xi_n(k+1) = \Theta_f^{\mathrm{T}}\Phi_{fn}(\xi_1(k)) + gu(k) \\ y(k) = \xi_1(k) \end{cases} \tag{5}$$

where the new parameters Θ_f and g as well as new system functions $\Phi_{fi}(\cdot)$ are defined as

$$\Theta_f = [\Theta_{f1}^{\mathrm{T}}, \ldots, \Theta_{fn}^{\mathrm{T}}]^{\mathrm{T}} \in R^p, \qquad \Theta_{fi} = \Theta_i\prod_{j=0}^{i-1}g_j$$

$$g = \prod_{j=0}^{n}g_j, \; \Phi_{fi}(\cdot) = \left[\mathbf{0}_{[M_i]}^{\mathrm{T}}, \Phi_i^{\mathrm{T}}(\cdot), \mathbf{0}_{[N_i]}^{\mathrm{T}}\right]^{\mathrm{T}} \in R^p$$

with $M_i = \sum_{j=1}^{i-1}p_j$, $N_i = \sum_{j=i+1}^{n}p_j$, $p = \sum_{j=1}^{n}p_j$. The transformed system (5) is very similar to the “parameter-output-feedback” form except that the control gain of $u(k)$ is an unknown constant g rather than one. The existence of the unknown control gain makes it impossible to calculate future values of the outputs by the approach proposed in Zhao and Kanellakopoulos (2002).