Automatica 45 (2009) 2187–2193

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

New algorithm for observer error linearization with a diffeomorphism on the outputs[☆]

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ARTICLE INFO

Article history:
Received 27 January 2007
Received in revised form
12 March 2009
Accepted 28 May 2009
Available online 19 August 2009

Keywords:
Nonlinear observers
Observable canonical form
Observer error linearization

ABSTRACT

In this paper, we give the necessary and sufficient conditions that guarantee the existence of a diffeomorphism which allows one to transform a multi-output nonlinear dynamical system into a normal observable canonical form. In particular, we propose an algorithm that permits one to derive such diffeomorphism. The main feature of the canonical form is that it is obtained by allowing a diffeomorphism on the outputs and it also allows one to design an observer with linear error dynamics. We first consider multi-output nonlinear dynamical system without inputs. We then extend the results obtained to multi-output nonlinear dynamical system with inputs.

1. Introduction

The problem of nonlinear observer design with linear error dynamics – which is sometimes referred to as the *observer error linearization problem* – has been the subject of study for many researchers (see e.g. Krener & Isidori, 1983; Xia & Gao, 1989 and references herein). The main approach to tackle this problem consists of finding a diffeomorphism that will permit one to transform a given dynamical system into a suitable observable canonical form that will allow one to design an observer which will possess a linear error dynamics.

This problem was first addressed by Krener and Isidori (1983) for single output dynamical systems of the following form:

$$\dot{x} = f(x) \tag{1}$$

$$y = h(x). \tag{2}$$

In effect, the authors provided necessary and sufficient conditions that guarantee the existence of a diffeomorphism $z = \phi(x)$ which permits one to transform system (1)–(2) into the following nonlinear observable canonical form

$$\dot{z} = (\phi_* f)(z) = Az + \beta(y) \tag{3}$$

$$y = h(\phi^{-1}(z)) = Cz \tag{4}$$

where $(\phi_* f)(x) = \mathrm{d}\phi(\phi^{-1}(z)).f(\phi^{-1}(z))$ and A and C are in the well-known Brunovsky or Companion observable form. Interestingly enough, the same proposed diffeomorphism $z = \varphi(x)$ linearizes the output equation (2) as well. In practice, however, one can allow some diffeomorphism on the output; thus generating a new output of the form $\bar{y} = \digamma(y)$ which might be a nonlinear function of the output y. In light of this, we have $\bar{y} = y$ in Eq. (4); that is, $\digamma = \mathbb{I}\mathrm{d}$ where $\mathbb{I}\mathrm{d}$ is the identity function, in this particular case. In fact, Krener and Respondek (1985) relaxed the linearization constraint on the output by enabling nonlinear diffeomorphisms on the output. More precisely, the authors provided the adequate sufficient conditions under which a multi-output nonlinear system of the form (1) and (2) is transformed into the following form

$$\dot{z} = Az + \beta(\bar{y}) \tag{5}$$

$$\bar{y} = \digamma(y). \tag{6}$$

On the other hand, Xia and Gao (1989) gave necessary and sufficient conditions to solve the observer error linearization problem for multi-output nonlinear systems in the case where $\digamma = \mathbb{I}\mathrm{d}$. The conditions proposed by Xia and Gao (1989) are characterized in terms of codistributions. Along the same lines, in the case where

[☆] The material in this paper was partially presented at NOLCOS 07. This paper was recommended for publication in revised form by Associate Editor Henri Huijberts under the direction of Editor Andrew R. Teel.

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doi:10.1016/j.automatica.2009.05.030

$\digamma = \mathbb{I}$d, one can find the work of Hou and Pugh (1999) for a geometrical characterization and by Phelps (1991) for algebraic characterization. Other related works on the topic dealt with the problem of the linearization of the error dynamics by the so-called generalized input-output injection or direct transformation (see e.g. Lopez, Plestan, and Glumineau (1999) and Plestan and Glumineau (1997)). In a similar fashion, other works dealt with the case where the above matrices A and C are of the Brunovsky form, depending on the output (see e.g. Guay (2002) and Respondek, Pogromsky, and Nijmeijer (2004)) with the aim of applying high gain observers (see e.g. Busawon, Farza, and Hammouri (1998)). A generalization of the work of Xia and Gao (1989) was done by Boutat, Zheng, Barbot, and Hammouri (2006) for a special form of the diffeomorphism $\digamma$.

In this paper, we are concerned with transforming multi-output systems of the form (1)–(2) into the form (5)–(6). We give necessary and sufficient conditions which permit such a transformation. As a result, we generalize the necessary and sufficient conditions given by Xia and Gao (1989) and Boutat et al. (2006) in the case where the diffeomorphism $\digamma$ is of a general form. However, by contrast to the conditions given in Xia and Gao (1989), the conditions proposed in this work are characterized in terms of Lie brackets. In addition, we present an algorithm for the computation of the diffeomorphism which allows one to transform (1)–(2) into (5)–(6). This is then used as a basis for the design of an observer for multi-output systems with linear error dynamics. We show, in particular, that the same computed diffeomorphism yields a diffeomorphism on the outputs. This computation is a necessary step to solve the so-called linearization problem by means of immersion. We then extend the results obtained to a multi-output nonlinear dynamical system with inputs. In particular, we extend the result obtained to dynamical systems that are not input affine. It is important to note that such a class of systems were not treated before with regards to the problem considered. This work is, in essence, a development of the work given in Boutat et al. (2006) and Boutat, Benali, and Hammouri (2007).

This paper is organized as follows: In the next section, we give the notations, the problem statement and some known results in the proposed topic. In Section 3, we present a technical result which is the key to proving the main result provided in Section 4. Section 5, is devoted to the case of multi-output nonlinear dynamical system with inputs. Finally, some conclusions are drawn on the various results obtained.

2. Notations and problem statement

In this section, we shall state the main problem under investigation and lay out the assumptions and notations on the class of systems considered throughout the paper.

We consider a multi-output nonlinear dynamical system described by:

$$\dot{x} = f(x) \tag{7}$$

$$y = h(x) \tag{8}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n, f : \mathcal{X} \subset \mathbb{R}^n \to \mathbb{R}^n$ and $h : \mathcal{X} \subset \mathbb{R}^n \to \mathbb{R}^m$. Without loss of generality, we assume that $0 \in \mathcal{X}$ and that $f(0) = 0$ and $h(0) = 0$. Furthermore, we assume that the components $y_1 = h_1, \ldots, y_m = h_m$ of the output h are independent.

Let $r_1, \ldots, r_m$ denote the observability indices of the pair (f, h); that is $(r_1, \ldots, r_m)$ is the m-uplet of integers defined as: $r_i = card\{m_j \geq i, j \geq 0\}$ where $m_j = \text{rank}(D_j) - \text{rank}(D_{j-1})$ with $D_k = \text{span}\{dL_f^j h_i; 0 \leq i \leq m, 0 \leq j \leq k\}$ and $m_0 = m = \text{rank}(D_0)$ (see e.g. Marino & Tomei, 1995).

We shall henceforth make the following assumptions:

A1. The observability indices $(r_i)_{1\leq i\leq m}$ of the dynamical system (7) and (8) are constant on X.

A2. The pair $(f(x), h(x))$ satisfies the observability rank condition. More specifically, the rank of the codistribution

$$\Delta = \text{span}\left\{\theta_{j,k}; j = 1, \ldots, m, k = 1, \ldots, r_j\right\} \tag{9}$$

is equal to $n = \sum_{i=1}^m r_i$ where

$$\theta_{j,k} = dL_f^{k-1} h_j. \tag{10}$$

Under the above assumptions, the problem in which we are interested in this work is stated as follows.

Problem 1. Consider a dynamical system described by (7)–(8) which satisfies Assumptions (A1) and (A2). Determine the necessary and sufficient conditions which guarantee the existence of a diffeomorphism $\phi(x) = z = (z_{1,1}, \ldots, z_{1,r_1}, z_{2,1}, \ldots, z_{2,r_2}, \ldots, z_{m,1}, \ldots, z_{m,r_m})$ such that, in the new coordinate system,

(a) the dynamics of system (7) is transform into

$$\dot{z} = \sum_{j=1}^{m}\left(\sum_{k=1}^{r_j-1} z_{j,k}\frac{\partial}{\partial z_{j,k+1}}\right) + \beta\left(z_{1,r_1}, \ldots, z_{m,r_m}\right) \tag{11}$$

(b) the variables $z_{1,r_1}, \ldots, z_{m,r_m}$ depend triangularly on the original outputs variables $(y_1, \ldots, y_m)$ as follows:

$$\bar{y} = \begin{pmatrix} z_{1,r_1} \\ z_{2,r_2} \\ \vdots \\ z_{m,r_m} \end{pmatrix} = \begin{pmatrix} \digamma_1(y_1) \\ \digamma_2(y_1, y_2) \\ \vdots \\ \digamma_m(y_1, y_2, \ldots, y_m) \end{pmatrix} = \digamma(y) \tag{12}$$

where $\digamma = (\digamma_1, \ldots, \digamma_m)$ is a local diffeomorphism. That is, for all $1 \leq i \leq m$, we have $\frac{\partial \digamma_i}{\partial y_i} \neq 0$.

Remark 2. Note that, by denoting $z_i = \text{col}\left(z_{i,1}, z_{i,2}, \ldots, z_{i,r_i}\right)$; $i = 1, \ldots, m$, we can rewrite system (11)–(12) in terms of m— subsystems of the form:

$$\left.\begin{aligned} \dot{z}_i &= A_i z_i + \beta_i(\bar{y}) \\ \bar{y}_i &= C_i z_i \end{aligned}\right\} \quad i = 1, \ldots, m \tag{13}$$

where A_i and C_i are respectively the $(r_i \times r_i)$ and $(1 \times r_i)$ constant matrices of the Brunovsky form:

$$A_i = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}, \qquad C_i = \begin{pmatrix} 0, & \cdots, & 0 & 1 \end{pmatrix}.$$

In the new coordinate system, even though the output $\bar{y}$ is some function of the original outputs variables, the new outputs are seen as a linear function of the new states variables. Consequently, one can design an observer for each subsystems as:

$$\dot{\hat{z}}_i = A_i\hat{z}_i + \beta_i(\bar{y}) + K_i(\bar{y}_i - C_i\hat{z}_i), \tag{14}$$

where K_i is chosen such that $(A_i - K_i C_i)$ is stable, hence, obtaining an overall observer with linear error dynamics. More precisely,

$$\dot{e}_i = (A_i - K_i C_i)\, e_i, \quad i = 1, \ldots, m$$

where $e_i = z_i - \hat{z}_i$.

2.1. Some recalls and observations when $\digamma = \mathbb{I}$d

Hereafter, we will recall the results given in Xia and Gao (1989) which dealt with Problem 1 in the particular case where $\digamma = \mathbb{I}$d in Eq. (12).

First of all, define the following family of vector fields $\left(\tau_{i,1}\right)_{1\leq i\leq m}$ such that: