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Brief paper

Receding horizon control of switching systems[☆]

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ABSTRACT

This paper exploits the concept of periodic invariance in order to derive a novel predictive control algorithm for switched systems. The offline computation here concerns the definition of sequences of switches which return the state vector to a set after a given number of moves, while an online optimization is used to improve performance by exploiting information available on the value of the state at each time instant. The algorithm is shown to have closed loop stability and to ensure that the state is steered to a bounded set of ellipsoids centered on the target state. The results of the paper are illustrated by means of a numerical example.

1. Introduction

In this paper, we provide a novel approach to Receding Horizon Control (RHC) for Switched Systems. The system dynamics can be described by a number of linear models, one for each state of a switching component. For example, two different linear state space models can be obtained for the case of a switched system comprising a single switching component with two states, e.g. an "on" and an "off" state. A typical example of such switched system is a dc–dc power converter. This kind of switching systems can be considered as a special case of hybrid systems which could be described in different frameworks (Borrelli, Baotic, Bemporad, & Morari, 2005; Branicky, Borkar, & Mitter, 1998; Heemels, De Schutter, & Bemporad, 2001).

RHC has the distinctive advantage that it performs, in a receding horizon manner, online optimizations which take account of constraints: explicitly over a finite prediction horizon; implicitly thereafter, through the use of a terminal set which is invariant under a fixed terminal state feedback control law, e.g. $\mathbf{u} = K\mathbf{x}$ (Mayne & Michalska, 1993). The concept of invariance can be overly restrictive and larger sets can be obtained through the use of periodic-invariance instead (Lee & Kouvaritakis, 2006; Lee, Kouvaritakis, & Cannon, 2009) according to which the state is allowed to be steered outside the set Ω so long as it can be steered back to Ω through a sequence of predefined state feedback control moves, $\mathbf{u} = K_i\mathbf{x}_i$, $i = 0, 1, 2, \ldots, N-1$.

In Borrelli et al. (2005) and Morari and Baric (2006), explicit RHC laws have been proposed for hybrid systems. With the explicit RHC design, the stability guaranteeing terminal constraint is omitted during the off-line optimization stage in order to reduce the conservativeness and complexity of the controller design. Instead, the closed-loop stability is checked after a Piecewise Affine (PWA) state feedback law is computed from the off-line optimization. If the resulting PWA state feedback is found to be unstable, one should carry out the off-line optimization again with different design parameters. The objective of the on-line part of the explicit RHC is to evaluate the PWA state feedback law according to the current state. The explicit RHC was applied to a dc–dc converter (Beccuti, Papafotiou, Frasca, & Morari, 2007a,b) by transforming the state average model of the dc–dc converter to a multi-step hybrid model. The resulting control law is given as $\mathbf{u} = K_i\mathbf{x}$ to determine the duty ratio of a PWM (Pulse Width Modulation) control in Beccuti et al. (2007a,b).

In the literature, two main approaches for the feedback regulation of dc–dc power converters are the PWM feedback strategy (Sira-Ramirez, Perez-Moreno, Ortega, & Garcia-Esteban, 1997) and the stabilizing sliding regimes method (Utkin, Guldner, & Shi, 1999). The sliding mode methods are known to be better than PWM controllers in respect of robustness to large-signal perturbations (Tan, Lai, Tse, & Cheung, 2005). The nature of sliding mode control, however, is to operate ideally at infinite switching frequencies to make the controlled variables track a certain reference path. Both the PWM and sliding mode controllers are developed in continuous time domain. The key problem of sliding mode

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approaches concerns the confinement of the switching frequency within an affordable limit while suppressing the chattering phenomenon.

This paper exploits the benefits of periodic invariance which, however, is introduced into the finite horizon of the first N predicted moves in order to derive an effective means of controlling the switched systems. In this context, the concept of periodic invariance can be applied to switched systems in such a way that a set Ω around a target is periodically invariant with respect to a particular switching sequence. Thus, here the system state, starting from the set Ω, is steered back to the set through the use of a sequence of N switchings. Periodic invariance provides a proof of closed loop stability through the use of results that guarantee contraction to a bounded ellipsoidal set. In addition, online optimization of the predicted switching sequence can accelerate convergence (to the origin, or the bounded set) and thus enhances performance. The results of the paper are illustrated by simulations performed on a model of a simple dc–dc power converter problem. Unlike earlier RHC methods on the control of dc–dc converter (Beccuti et al., 2007a,b), it does not require averaged models of the switched systems and can be readily applied to dc–dc converters with multiple switching components.

2. Problem formulation

Consider a switched system having one switching component; extension of the results of the paper to the general case is straightforward and is omitted for the sake of clarity of presentation. Depending on whether the switch is on or off, the dynamics of such a system are described by:

$$\text{Switch on: } \mathbf{x}_{k+1} = A_1\mathbf{x}_k + B_1\mathbf{w} \tag{1}$$

$$\text{Switch off: } \mathbf{x}_{k+1} = A_2\mathbf{x}_k + B_2\mathbf{w} \tag{2}$$

where $\mathbf{x} \in R^{n\times 1}$ is the state vector and $\mathbf{w}$ is the strength of a constant voltage (or current) source. This kind of formulation can be applied to many types of switching power converters. The control objective is to select the switching sequence so as to maintain $\mathbf{x}$ as close as possible to a given reference state, χ, defined by two virtual system models:

$$\chi = A_{v_1}\chi + B_1\mathbf{w} \tag{3}$$

$$\chi = A_{v_2}\chi + B_2\mathbf{w} \tag{4}$$

for which χ is clearly an equilibrium state. Subtraction of (3)–(4) from (1)–(2), respectively, gives:

$$\text{Switch on: } \mathbf{z}_{k+1} = A_1\mathbf{z}_k + (A_1 - A_{v1})\chi \tag{5}$$

$$\text{Switch off: } \mathbf{z}_{k+1} = A_2\mathbf{z}_k + (A_2 - A_{v2})\chi \tag{6}$$

where $\mathbf{z}_k := \mathbf{x}_k - \chi$ denotes the error state vector. In this setting, the control objective is equivalent to the selection of switching between the on/off states so as to make the error state, $\mathbf{z}_k$, as small as possible. The purpose of this paper is to propose a novel strategy for meeting this objective.

3. Switching and periodic invariance

For the case considered here (i.e. for a single switching component with two switching states (on/off) over a horizon of N predicted switchings) there is a set Σ of 2^N possible predicted switching sequences (comprising N elements each assuming the values of 1 or 2). Over Σ it is possible to extend the concept of periodic invariance:

Definition 1. A set E_o is defined to be periodically invariant with respect to Σ if there exists a switching sequence $S \in \Sigma$ which steers all initial states $\mathbf{x}_k \in E_o$ to an end state $\mathbf{x}_{k+N} \in E_o$.

The definition above does not imply contraction. However, in the sequel, conditions will be given under which repeated application of the switching sequence specified in Definition 1 would result in contraction for initial conditions which are sufficiently far from the reference state, and that this process of contraction carries on until the state comes to be sufficiently close; the meaning of both the terms "sufficiently far" and "sufficiently close" to the reference state will be made precise through the statement of the relevant results given later in this section. Thus, periodic invariance, hereafter abbreviated as PI, can be used as the basis for the derivation of closed loop practical stability. However before the presentation of the stability results, PI conditions are derived for the case of ellipsoidal sets:

$$E_j := \left\{\mathbf{z}|\mathbf{z}^{\mathrm{T}}P_j\mathbf{z} \le a_j\right\}, \quad P_j > 0, j = 0, 1, \ldots, N-1. \tag{7}$$

This will be achieved through the use of the augmented ellipsoids defined on an augmented state vector $\hat{\mathbf{z}} := [\chi^{\mathrm{T}}\ \mathbf{z}^{\mathrm{T}}]^{\mathrm{T}}$:

$$\hat{E}_j := \left\{\hat{\mathbf{z}}|\hat{\mathbf{z}}^{\mathrm{T}}\hat{P}_j\hat{\mathbf{z}} \le b_j\right\}, \quad j = 0, 1, \ldots, N-1, \tag{8}$$

where $\hat{P}_j := \mathrm{diag}(\Pi_j, P_j)$ with Π_j, P_j being n-dimensional positive definite matrices, and $b_j := a_j + \chi^{\mathrm{T}}\Pi_j\chi$. The dynamics of the augmented state are described by

$$\hat{\mathbf{z}}_{j+1} = \tilde{A}_j\hat{\mathbf{z}}_j, \quad j = 0, 1, \ldots, N-1, \tag{9}$$

where, depending on whether the jth element of the switching sequence S is 1 or 2, the matrix $\tilde{A}_j$ assumes one of the two values, respectively:

$$\hat{A}_1 := \begin{bmatrix} I & 0 \\ A_1 - A_{v1} & A_1 \end{bmatrix} \qquad \hat{A}_2 := \begin{bmatrix} I & 0 \\ A_2 - A_{v2} & A_2 \end{bmatrix}. \tag{10}$$

From (9), it is easy to see that:

$$\hat{P}_j - \tilde{A}_j^{\mathrm{T}}\hat{P}_{j+1}\tilde{A}_j > 0; \quad j = 0, 1, \ldots, N-1, \tag{11}$$

implies that $\hat{\mathbf{z}}_{j+1}^{\mathrm{T}}\hat{P}_{j+1}\hat{\mathbf{z}}_{j+1} < \hat{\mathbf{z}}_j^{\mathrm{T}}\hat{P}_j\hat{\mathbf{z}}_j$ so that through a recursive use of (11) for $j = 0, 1, \ldots, N-1$ it follows that $\hat{\mathbf{z}}_N^{\mathrm{T}}\hat{P}_N\hat{\mathbf{z}}_N < \hat{\mathbf{z}}_0^{\mathrm{T}}\hat{P}_0\hat{\mathbf{z}}_0$, which can be rearranged as:

$$\mathbf{z}_N^{\mathrm{T}}P_N\mathbf{z}_N < \mathbf{z}_0^{\mathrm{T}}P_0\mathbf{z}_0 + \beta_0, \tag{12}$$

where $\beta_0 := \chi^{\mathrm{T}}(\Pi_0 - \Pi_N)\chi$. These conditions will be used, below, to establish the fact that recursive use of a switching sequence which satisfies (11) results in contraction for an initial condition which is "sufficiently far" from the reference state (as stated in Theorem 1) and leads to periodic invariance for initial conditions which are "sufficiently near" to the reference state (Theorem 2).

Theorem 1. *Let there exist a switching sequence S and matrices $\hat{P}_j$ such that the LMI conditions*

$$\begin{bmatrix} \hat{P}_j & (\hat{P}_{j+1}\tilde{A}_j)^{\mathrm{T}} \\ \hat{P}_{j+1}\tilde{A}_j & \hat{P}_{j+1} \end{bmatrix} > 0, \quad j = 0, \ldots, N-1 \tag{13}$$

hold true with $P_j, \Pi_j > 0$, where $\tilde{A}_j$ is determined to be $\hat{A}_1$ or $\hat{A}_2$ depending on the j^{th} element of the switching sequence S. If there exists $0 < \gamma < 1$ satisfying

$$P_N \ge \frac{1}{\gamma}P_0 \tag{14}$$

then, for initial conditions which are "sufficiently far" from the reference state χ in the sense that

$$\mathbf{z}_0^{\mathrm{T}}P_0\mathbf{z}_0 > \frac{\gamma\beta_0}{1-\gamma}, \tag{15}$$

application of the switching sequence S ensures the contraction condition

$$\mathbf{z}_N^{\mathrm{T}}P_0\mathbf{z}_N < \mathbf{z}_0^{\mathrm{T}}P_0\mathbf{z}_0. \tag{16}$$

Proof. On account of (11), which is equivalent to (13), $\mathbf{z}_N$ must satisfy (12) and over all $\mathbf{z}_N$ that satisfy (12) the $\mathbf{z}_N$ that maximizes $\mathbf{z}_N^{\mathrm{T}}P_0\mathbf{z}_N$, say $\mathbf{z}_N^*$, must be such that (12) holds with equality. Hence