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Brief paper

Fault Detection and Isolation of discrete-time Markovian jump linear systems with application to a network of multi-agent systems having imperfect communication channels[☆]

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ABSTRACT

This paper deals with the problem of Fault Detection and Isolation (FDI) for discrete-time Markovian Jump Linear Systems (MJLSs). A geometric property related to the unobservable subspace of an MJLS is first presented and the concept of unobservability subspace is introduced. Sufficient conditions for designing an H_∞-based FDI algorithm for MJLSs subject to input disturbances and measurement noise are presented and developed. Our proposed approach is then applied to the problem of fault detection and isolation in a network of multi-agent systems when imperfect communication channels exist among the agents. A discrete-time communication link with a stochastic packet dropping effect is considered based on the Gilbert–Elliott model and the entire network is modeled as a discrete-time MJLS. Simulation results are presented for formation flight of satellites to demonstrate and verify the effectiveness and performance capabilities of our proposed FDI algorithm.

1. Introduction

A great deal of attention has recently been devoted to the Markovian jump linear systems (MJLSs) (Boukas, 2006; Cao & Lam, 2000; Costa, Fragoso, & Marques, 2005; de Souza & Fragoso, 2002; Shi, Boukas, & Agarwal, 1999; Xing & Lam, 2006) which comprise an important class of hybrid systems. This family of systems is generally modeled by a set of linear systems with transitions between models that are determined by a Markov chain taking values in a finite set. Markovian jump systems are popular in modeling many practical systems where one may experience abrupt changes in system structure and parameters. These changes are quite common and do frequently occur in manufacturing systems, economic systems, communication systems, power systems, etc. (Mariton, 1990). Recently, Markovian jump systems have also gained interest for their capability in modeling behaviors and phenomenon in networks that consist of sensors, actuators and processors (Gupta, Murray, & Hassibi, 2003; Huang & Dey, 2007; Jin, Gupta, & Murray, 2006).

In recent years, only a few works on fault detection and isolation of MJLSs have appeared in the literature. In Zhang, Ding, Frank, and Sader (2004), Zhong, Ye, Shi, and Wang (2005), Mao, Jiang, and Shi (2007), a robust fault detection (and *not an isolation*) filter for a discrete-time MJLS is developed based on an H_∞ filtering framework, in which the residual generator is also an MJLS. An LMI approach is developed for solving the problem. In Wang, Wang, Gao, and Wu (2006), a robust fault identification filter for a class of discrete-time MJLSs with mode dependent time delays and norm bounded uncertainty is developed based on an H_∞ optimization technique. In Wang et al. (2006), the generated residual signal is an estimate of the fault signal. In Meskin and Khorasani (2009b), the fundamental problem of fault detection and isolation for continuous-time MJLSs is solved based on a geometric approach. However, the problem of fault isolation for discrete-time MJLSs has not been completely solved and fully addressed in the above references.

In this paper, a geometric approach is adopted for the FDI problem of discrete-time MJLSs. Towards this end, the first contribution of this paper is the derivation of a geometric property for the unobservable subspace of discrete-time MJLSs. The notion of an unobservability subspace is then introduced for MJLSs and an algorithm for constructing the smallest unobservability subspace containing a given subspace is proposed. By utilizing the developed geometric framework, the problem of designing an H_∞-based FDI algorithm for an MJLS that is subjected to external disturbances and noise is investigated. A preliminary version of these results was presented in Meskin and Khorasani (2008).

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Subsequently, our proposed FDI algorithm is applied to a network of multi-agent systems in the presence of imperfect communication links. As a case study, we consider the FDI problem for formation flight of satellites that are operating under imperfect communication links with packet dropout among the satellites. The packet delivery characteristics of the communication network can be modeled as a Bernoulli process or a two-state Markov process (Elliott, 1963; Gilbert, 1960; Gupta et al., 2003; Jin et al., 2006). We consider the latter model which is commonly used for modeling the fading communication channels and is also known as the *packet erasure channel model*. Based on the Gilbert–Elliott model, a network of multi-agent systems is integrated with a two-state Markov process model of the communication channel which is then represented as an overall discrete-time Markovian jump system. The FDI problem in a network of multi-agent systems in the presence of imperfect communication links is then solved in the framework of Markovian jump systems.

To summarize, the main contributions of this work are as follows:

(1) Development of a theoretical foundation based on a geometric framework for simultaneously solving the problems of robust fault detection and fault isolation of discrete-time MJLSs.
(2) Development of an H_∞-based structured FDI algorithm for discrete-time MJLSs.
(3) Development of an FDI algorithm for a network of multi-agent systems in the presence of imperfect communication links.
(4) Application of the above developed theoretical results to the FDI problem of formation flight of satellites subject to imperfect communication links.

The remainder of this paper is organized as follows. In Section 2, the geometric characteristics of unobservable subspaces for MJLSs is developed and the notion of an unobservability subspace is formally introduced. An H_∞-based fault detection and isolation strategy for MJLSs is developed in Section 3. Finally, in Section 4, our proposed algorithm is applied to a network of multi-agent systems in the presence of imperfect communication links. Simulation results are presented for formation flight of satellites in Section 5. Conclusions and future work are presented in Section 6.

The following notation is used throughout this paper. Script letters $\mathcal{X}, \mathcal{U}, \mathcal{Y}, \ldots,$ denote real vector spaces. For any positive integer k, $\mathbf{k}$ denotes the finite set $\{1, 2, \ldots, k\}$. A subspace $\mathcal{S} \subseteq \mathcal{X}$ is termed A-invariant if $A\mathcal{S} \subseteq \mathcal{S}$. For A-invariant subspace $\mathcal{S} \subseteq \mathcal{X}$, $A : \mathcal{S}$ denotes the restriction of A to $\mathcal{S}$, and $A : \mathcal{X}/\mathcal{S}$ denotes the map induced by A on the factor space $\mathcal{X}/\mathcal{S}$. For given maps $A_i, i \in \Psi$ and a subspace $\mathcal{K}$, the largest A_i-invariant subspace $i \in \Psi$ that is contained in $\mathcal{K}$ is denoted by $\ll \mathcal{K}|A_i \gg_{i\in\Psi}$. We denote by $\|.\|$ the standard norm in $\mathbb{R}^n$. The symbol $*$ within a matrix represents its symmetric elements. $\otimes$ denotes the Kronecker product and I_N is an $N \times N$ identity matrix.

2. Unobservable and unobservability subspaces

In this section, a geometric definition for an unobservable subspace of discrete-time MJLSs is introduced. The notion of unobservability subspace is then formalized for discrete-time MJLSs that are governed by (1) and an algorithm for constructing the smallest unobservability subspace containing a given subspace is proposed. As shown in the next section, the unobservability subspace for an MJLS plays a central role in solving the fault detection and isolation problem. Consider the discrete-time MJLS written as

$$\begin{aligned} x(k+1) &= A_{\lambda(k)}x(k) \\ y(k) &= C_{\lambda(k)}x(k), \quad x(0) = x_0, \lambda(0) = i_0 \end{aligned} \tag{1}$$

where $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ are the state and output variables with dimensions n and q, respectively; and $\{\lambda(k), k \geq 0\}$ is a discrete-time irreducible Markov process taking values in the finite set $\Psi = \{1, \ldots, N\}$. The Markov process describes the switching between the different system modes and its evolution is governed by the probability transitions $\pi_{ij} = Pr\{\lambda(k+1) = j|\lambda(k) = i\}$, where $\sum_{j=1}^{N} \pi_{ij} = 1$. It is assumed that $\pi_{ii} > 0$, $i \in \Psi$. The matrices $A_{\lambda(k)}$ and $C_{\lambda(k)}$ are known constant matrices for all $\lambda(k) = i \in \Psi$. For simplicity, we denote the matrices associated with $\lambda(k) = i$ by $A_{\lambda(k)} = A_i$ and $C_{\lambda(k)} = C_i$. We first start with the definition of weak observability for the MJLS (1).

Definition 1 (*Costa and do Val (2002)*). The system (1) is said to be weakly ($W-$) observable when there exists a $\gamma > 0$ such that $W^{n^2N}(x_0, i_0) \geq \gamma|x_0|^2$ with

$$W^{n^2N}(x_0, i_0) = E\left\{x_0^\top \Gamma(n^2N)x_0|x(0) = x_0, \lambda_0 = i_0\right\}$$

$\forall x_0 \in \mathcal{X},\ i_0 \in \Psi$, where the observability Grammian Γ is defined as $\Gamma(k) = \sum_{t=0}^{k} \Phi^\top(t)C_{\lambda(t)}^\top C_{\lambda(t)}\Phi(t)$ and Φ is the state transition matrix of system (1).

In Costa and do Val (2002), a collection of matrices $\mathcal{O} = \{\mathcal{O}_1, \ldots, \mathcal{O}_N\}$ is introduced for testing the W-observability of MJLSs according to the following procedure. Let $O_i(0) = C_i^\top C_i,\ i \in \Psi$ and define the sequence of matrices as follows

$$O_i(k) = A_i^\top\left(\sum_{j=1}^{N}\pi_{ij}O_j(k-1)\right)A_i \quad k > 0, i \in \Psi. \tag{2}$$

Next the matrix $\mathcal{O}_i$ is defined according to

$$\mathcal{O}_i = [O_i(0)\ O_i(1)\cdots O_i(n^2N-1)]^\top. \tag{3}$$

It is shown in Costa and do Val (2002) that MJLS (1) is W-observable if and only if $\mathcal{O}_i$ has a full rank for each $i \in \Psi$. Moreover, a state x is said to be unobservable if $W^t(x, i) = 0$ for all $i \in \Psi$. Let $\mathcal{Q}$ denotes the unobservable set for MJLS (1), i.e.

$$\mathcal{Q} = \{x|W^t(x, i) = 0, \forall i \in \Psi, t \geq 0\}. \tag{4}$$

It is shown in Costa and do Val (2002) (Lemma 3) that for irreducible Markov processes, $\mathcal{N}\{\mathcal{O}_i\} = \mathcal{N}\{\mathcal{O}_j\}$, $i, j \in \Psi, i \neq j$ and $\mathcal{Q} = \mathcal{N}\{\mathcal{O}_i\}$. Therefore, $\mathcal{Q}$ is the subspace of $\mathcal{X}$ and is called the unobservable subspace of MJLS (1). Our first result introduced below characterizes a geometric property of $\mathcal{Q}$.

Theorem 2. *An unobservable subspace $\mathcal{Q}$ for MJLS (1) is the largest A_i-invariant ($i \in \Psi$) that is contained in $\mathcal{K} = \bigcap_{i=1}^{N} \mathrm{Ker}C_i$, i.e. $\mathcal{Q} = \ll \mathcal{K}|A_i \gg_{i\in\Psi}$.*

Proof. It follows from the above discussion that $\mathcal{Q} \subseteq \mathrm{Ker}C_i,\ i \in \Psi$, and hence $\mathcal{Q} \subseteq \mathcal{K}$. Let $x \in \mathcal{Q}$. Our goal is to show that $A_ix \in \mathcal{Q}$ for all $i \in \Psi$ (i.e. $\mathcal{Q}$ is A_i-invariant, $i \in \Psi$). Since $x \in \mathcal{N}\{O_i(k-1)\}$ and $x \in \mathcal{N}\{O_i(k)\},\ i \in \Psi$, then

$$x^\top O_i(k)x = x^\top A_i^\top\left(\sum_{j=1}^{N}\pi_{ij}O_j(k-1)\right)A_ix = 0$$

and $O_i(k-1)A_ix = 0$, since $\pi_{ii} > 0$. Hence, $A_ix \in \mathcal{N}\{O_i(k-1)\}$ for all $k > 0$ and $A_ix \in \mathcal{N}\{\mathcal{O}_i\}$ for all $i \in \Psi$. This shows that $\mathcal{Q}$ is A_i-invariant for all $i \in \Psi$.

Next we show that $\mathcal{Q}$ is the largest A_i-invariant ($i \in \Psi$) that is contained in $\mathcal{K}$. Let $\mathcal{V}$ be an A_i-invariant ($i \in \Psi$) subspace that is contained in $\mathcal{K}$. Clearly, $\mathcal{V} \subseteq \mathcal{N}\{O_i(0)\},\ i \in \Psi$. Let $\mathcal{V} \subseteq \mathcal{N}\{O_i(k-1)\},\ i \in \Psi$ and $x \in \mathcal{V}$, then $x^\top O_i(k)x = x^\top A_i^\top\left(\sum_{j=1}^{N}\pi_{ij}O_j(k-1)\right)A_ix = 0$ since $A_ix \in \mathcal{V}$ ($\mathcal{V}$ is A_i-invariant). Hence, $\mathcal{V} \subseteq \mathcal{N}\{O_i(k)\},\ i \in \Psi$. This shows that $\mathcal{Q}$ contains all the subspaces that are A_i-invariant ($i \in \Psi$) and are contained in $\mathcal{K}$. ■