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Analysis of bubble-in-liquid bridge configuration as prototype for studying foam dynamics. Zero Bond number case

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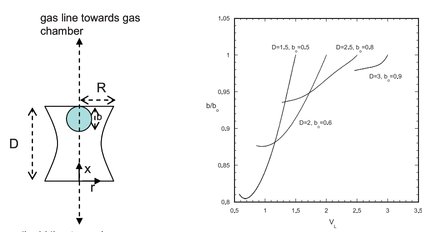
HIGHLIGHTS

- Theoretical study of bubble in liquid bridge problem for zero Bond number.
- Existence of two bridge rupture mode: neck rupture and film rupture.
- Existence of an optimum bubble size for maximizing bridge stability.

GRAPHICAL ABSTRACT



Bubble radius evolution in a liquid bridge undergoing drainage

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ABSTRACT

A bubble-in-liquid bridge is a unique configuration that resembles the case of two neighboring bubbles in a foam separated by a liquid layer. It calls for a small bubble inside a liquid bridge with the curved free surface of the liquid bridge acting as part of a larger external bubble. This configuration can serve as a prototype to study foam dynamics. A devise exploring this concept has been presented in a previous work (Kostoglou et al., 2011). The present work adds to the theoretical background of the proposed devise. The particular case of zero Bond number is studied theoretically here. Even in its simplest form the particular system exhibits an interesting behavior. It is shown that there are two rupture modes of the liquid bridge as the liquid volume decreases i.e. neck rupture and film rupture. The prevailing one depends on system parameters. The evolution of the bubble size as the liquid volume decreases up to rupture, is extensively studied.

1. Introduction

Liquid bridges formed between solid surfaces have been the study of extensive research through the last 100 years. The main reason is that apart of the fundamental importance of the subject, there are a variety of applications involving liquid bridges. For example, liquid bridges between particles are important in particulate processes such as granulation, flotation, coating [1,2]. They are also important for moist soil properties [3]. Another family of applications regards the production of high quality crystals [4], alignment of components using optoelectronics assemblies, and the control of forces in microgripping processes [5]. Study of the above processes requires the knowledge of the bridge shape which is dictated by the well-known Young–Laplace equation. In general, this equation does not have analytical solution so a large amount of literature is devoted to derivation of approximate solutions of this equation that offer physical insight and facilitate interpretation of experimental data [6–11].

Several designs based on quasi-static or dynamic liquid bridges have been proposed as tools for studying static or dynamic surface properties [12,13]. Traditionally, liquid bridges have been studied either by measuring the force exerted by the liquid bridge to its supporting boundaries or using image processing techniques to

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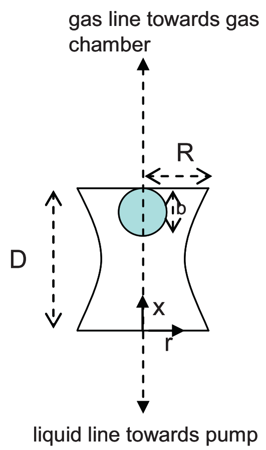

Fig. 1. Schematic of the liquid bridge-bubble system.

identify the liquid bridge profile. In both cases, a usual goal was to estimate surface tension or contact angle from liquid bridge characteristics. However, both techniques require excessive skill and are cumbersome. The apparent electrical conductance of conducting liquid bridges has been suggested as an alternative characteristic parameter from which liquid bridge features can be directly deduced [12,14]. Recently, a configuration consisting of a bubble-in-liquid bridge has been proposed to examine the surfactant induced stabilization of the liquid layer between the internal bubble and the external free surface of the bridge as the liquid was draining out of the bridge [14]. An advantage of this configuration is that the stability of the liquid layer can be followed continuously starting from an appreciable thickness of the layer down to a very thin film up to the rupture point. Ultra-sensitive electrical conductance measurements were employed to follow the drainage of a liquid bridge pinned at the rims of flat rod electrodes. Electrical signals can easily sense liquid films of the order of microns or less without the complications of optical distortion and focusing of traditional imaging techniques. The analysis performed in [14] was based on the assumption of a constant bubble size during liquid drainage. This assumption is relaxed in the present work by considering an evolving bubble size during drainage.

The scope of the present study is the fundamental analysis of the configurations of the bubble-in-liquid bridge system during liquid volume reduction (drainage of the bridge) in the absence of surfactant and for zero Bond number. In the next section the mathematical problem is formulated and the behavior of its solution with emphasis to the evolution of the bubble size is discussed in the results section.

2. Problem formulation

The geometry of the problem is shown in Fig. 1. A liquid bridge is edge-pinned between the tips of two equal diameter solid rods which are aligned vertically. Both rods have tiny holes at their center. These holes have diameters an order of magnitude smaller than the diameter of the rods. The hole at the top rod is meant to create an internal bubble to the bridge by blowing air through it whereas the hole at the bottom rod is meant for draining the bridge liquid. The internal bubble is connected to an air chamber. From the practical point of view the air chamber is necessary for creating the bubble and measuring is internal pressure. The pressure and volume in the chamber is adjusted to achieve a bubble of the desired size. The issue of interest here is to study the evolution of the liquid bridge shape, the bubble size and the pressure in the liquid during liquid withdrawal. The study will be performed for conditions that correspond to the following assumptions.

(i) The liquid is a pure liquid with surface tension γ. There are no surfactants in the system for surface tension manipulation. Surfactant through their adsorption/desorption on interfaces and diffusion adds significant complexity to the problem and cannot be taken into account by simply altering the equilibrium surface tension [15].
(ii) The whole process occurs under constant temperature conditions and 100% relative humidity to prevent evaporation.
(iii) The effect of gravity can be ignored (negligibly small value of the Bond number). This condition is met in three situations (based on different ways to decrease Bo): microgravity environment, small dimensions of the liquid bridge, replacing the external air by another immiscible liquid whose density matches the density of the bridge liquid.
(iv) Rod material fully wetted by the bridge liquid. This assumption combined with (i) and (iii) ensures that the shape of the internal bubble is always spherical.
(v) The gas is insoluble in the liquid. This assumption is needed to exclude the Ostwald ripening phenomenon i.e. the bubble dissolution due to its higher pressure than the environment pressure.
(vi) The liquid withdrawal flowrate is relatively small to ensure that the viscous stresses are negligibly small compared to the surface tension forces and the bridge shape can be determined by pseudo-steady energy balance (i.e. Young–Laplace equation). This assumption implies that the liquid flow rate does not alter the evolution path of the system but simply influences the time variable. The state of the system is fully determined by its liquid content which can be used as the time-like evolution variable.

Considering a cylindrical coordinate system x, r with its center at the center of the bottom rod and denoting as R the rod radius, D the liquid bridge length (height), V_{Lo} the initial liquid volume, V_{L} the instantaneous liquid volume, b_{o} the initial internal bubble radius, b the instantaneous internal bubble radius, V_{c} the volume of the air chamber connected with the bubble, P_{o} the environmental pressure and $r = Y(x)$ the shape of the liquid bridge, the problem is described from the following set of equations:

Young–Laplace equation for the liquid bridge shape

$$\left(1+\left(\frac{dY}{dx}\right)^2\right)^{-3/2}\left[-\frac{d^2Y}{dx^2}+\frac{1}{Y}\left(1+\left(\frac{dY}{dx}\right)^2\right)\right]=\frac{\Delta P}{\gamma} \tag{1}$$

where $\Delta P = P_{\mathrm{L}} - P_{\mathrm{o}}$ is the pressure difference between the liquid bridge and the environment due to the curvature of the liquid bridge. The boundary conditions for the above second order boundary value problem are

$$Y(0) = R, \tag{2a}$$

$$Y(D) = R \tag{2b}$$

The additional unknown parameter ΔP is found from the requirement of liquid volume conservation:

$$V_L = \pi\int_0^D [Y^2(x) - U(b^2 - (x - D + b)^2)]\,dx \tag{3}$$

where the function U is defined as $U(x) = 0$ for $x < 0$ and $U(x) = x$ for $x \geq 0$.

The above mathematical problem can be solved for the instantaneous liquid bridge shape for a given bubble diameter b. An evolution equation for the bubble radius is needed and it is based on the requirement that the mass of gas in the domain defined by the