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Brief paper

Bearings only single-sensor target tracking using Gaussian mixtures[☆]

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ABSTRACT

This paper presents a new approach for single sensor tracking using passive bearings only measurements. Gaussian mixture measurement presentation, together with a track splitting algorithm, allow space-time integration of the target position uncertainty with a simple algorithm. The bearings-only measurements are incorporated into track as they arrive using a dynamic bank of linear Kalman filters. While this approach is applicable to the case with the target detection, data association and multitarget issues, this paper concentrates on the target trajectory estimation using associated measurements. A simulation study demonstrates the benefits of this approach.

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1. Introduction

For many decades bearings only sensors have been used for target (emitter) localization. If multiple sensors are used, the emitter is located by multiple beam crossing (triangulation) (Bar-Shalom & Li, 1993; Mušicki, 2008b). Movement from a single sensor can also provides effective triangulation. The necessary observability condition is that (at least in some interval) the sensor motion model must be at least one derivative higher than the target motion, and a component of this motion must be perpendicular to the line of sight (Blackman & Popoli, 1999; Nardone, Lindgren, & Gong, 1984). If the target motion is a constant velocity, at some stage the sensor has to accelerate.

Single sensor single target bearings only tracking is a non-linear problem. Extended Kalman filter (EKF) (Bar-Shalom, Li, & Kirubarajan, 2001) linearizes nonlinearities around the predicted target position. Unscented Kalman filters (UKF) (Julier & Uhlmann, 2004; Julier, Uhlmann, & Durrant-Whyte, 2000) sample and propagate the probability density function at sigma points. Particle Filters (PF) (Ristic, Arulampalam, & Gordon, 2004) sample non-linear pdf by a set of random particles. EKF (Aidala, 1979), a bank of EKFs (Kronhamn, 1998; Peach, 1995), UKF (Ristic et al., 2004) and Particle Filters (Ristic et al., 2004) have all been applied to this problem. The Shifted Rayleigh filter (Arulampalam, Clark, & Vinter, 2007) is a moment matching algorithm developed specifically for this application; it generates the exact conditional distribution of the target motion, given normal approximation to the prior.

The Gaussian Mixture Measurements-Integrated Track Splitting (GMM-ITS) filter is an approach to estimation using nonlinear measurements (Mušicki & Evans, 2006). Both non-Gaussian measurement likelihood and non-Gaussian track state are approximated by Gaussian mixtures. This presentation is similar to the track splitting approach (Mušicki, La Scala, & Evans, 2007), and is a dynamic bank of linear Kalman filters. In this paper we evaluate the state estimation capabilities of GMM-ITS when used in single sensor bearings only tracking of a single target. This framework can easily be extended to data association issues when the measurement set contains clutter measurements, and targets exist and are detected randomly. It is used in Mušicki (2008a,b) to solve a different problem of asynchronous triangulation.

This approach has some similarities to Kronhamn (1998), where the track state is a static bank of Extended Kalman filters, initialized by Gaussian mixture approximation of the measurement likelihood. The approximation of measurement likelihood presented in Section 3.1 is very similar to solution (Kronhamn, 1998).

To summarize, the contributions of this paper are:

- Present a solution of single sensor bearings only tracking, using recently published GMM-ITS.
- GMM-ITS is a general non-linear estimator. It allows a trade-off between computational requirements and system performance, in this case estimation errors.
- The solution is compared with a number of other proposed solutions, and with the Cramer–Rao lower bound.

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The single sensor passive bearings only problem is presented in Section 2. Section 3 presents GMM-ITS solution. A simulation study presented in Section 4 shows the effectiveness of this method, followed by concluding remarks.

2. The single sensor bearings only tracking problem

A two-dimensional measurement case is considered. A single moving sensor measures direction of target emissions at known random times indexed by k. At some point the target trajectory becomes observable (Blackman & Popoli, 1999; Nardone et al., 1984). The target trajectory is modeled by

$$t_k = F_{k-1}t_{k-1} + \nu_{k-1} \tag{1}$$

where t_k denotes the target trajectory state at time k, F_{k-1} is the state propagation matrix, process noise ν_{k-1} is a zero mean, white Gaussian sequence with covariance Q_{k-1}. F_{k-1} and Q_{k-1} depend on the time between k and $k-1$ (Bar-Shalom & Li, 1993). Denote by $z_{k,s}$ sensor position at time k. Define target polar coordinates (r_k, θ_k) by

$$Ht_k - z_{k,s} = r_k \begin{bmatrix}\cos(\theta_k)\\ \sin(\theta_k)\end{bmatrix} \tag{2}$$

where Ht_k is the (linear) projection of target state into surveillance (position) space. The target range uncertainty is defined by a known minimum and maximum distance to sensor, $r_{k,\min} \le r_k \le r_{k,\max}$. Thus $\theta_k = \theta_k(Ht_k; z_{k,s})$ is a function of unknown target position Ht_k with sensor position $z_{k,s}$ being the parameter. Measurement $\theta_{k,m}$ is

$$\theta_{k,m} = \theta_k + \omega_k, \tag{3}$$

where ω_k is zero mean, white Gaussian measurement noise with covariance $\sigma^2_{k,\theta}$, uncorrelated with the process noise sequence. Probability density function $p(\theta_{k,m}|\theta_k)$ is

$$p(\theta_{k,m}|\theta_k) = \mathcal{N}(\theta_{k,m}; \theta_k, \sigma^2_{k,\theta}), \tag{4}$$

where $\mathcal{N}(x; m, \Sigma)$ denotes the Gaussian pdf of variable x with mean m and covariance Σ. We also use θ^k_m to denote the set of all measurements up to time k

$$\theta^k_m = \{\theta_{1,m}, \theta_{2,m}, \ldots, \theta_{k,m}\}. \tag{5}$$

3. GMM-ITS bearings only target tracking

A posteriori state estimate probability density function (pdf) is calculated by the Bayes formula, which here becomes

$$p(t_k|\theta^k_m) = \frac{p(\theta_{k,m}|t_k)p(t_k|\theta^{k-1}_m)}{p(\theta_{k,m}|\theta^{k-1}_m)}. \tag{6}$$

Measurement $\theta_{k,m}$ is known; measurement likelihood $p(\theta_{k,m}|t_k)$ is a function of target position Ht_k, $p(\theta_{k,m}|Ht_k) = f_m(Ht_k; \theta_{k,m})$. Eq. (6) can be expressed as

$$p(t_k|\theta^k_m) = \frac{f_m(Ht_k; \theta_{k,m})p(t_k|\theta^{k-1}_m)}{\int_{t_k} f_m(Ht_k; \theta_{k,m})p(t_k|\theta^{k-1}_m)\, \mathrm{d}t_k}. \tag{7}$$

Function $f_m(t_k; \theta_{k,m})$ is non-negative for all t_k, and can integrate to any positive number. Thus it is a density of t_k, and multiplying it by a constant does not change result (7). Here we choose to normalize it so that

$$\int_{t_k} f_m(Ht_k; \theta_{k,m})\, \mathrm{d}(Ht_k) = 1 \tag{8}$$

and $f_m(Ht_k; \theta_{k,m})$ is interpreted as target position Ht_k pdf, given $\theta_{k,m}$, and in the absence of any other information.

As $\theta_k(Ht_k; z_{k,s})$ is a non-linear function of Ht_k (2), neither $f_m(Ht_k; \theta_{k,m})$, nor $p(t_k|\theta^k_m)$ nor (for $k > 1$) $p(t_k|\theta^{k-1}_m)$ are Gaussian. GMM-ITS approximates both $f_m(Ht_k; \theta_{k,m})$ and $p(t_k|\theta^{k-1}_m)$ by Gaussian mixtures.

3.1. Measurement likelihood Gaussian mixture

The normalized measurement likelihood equals

$$f_m(Ht_k; \theta_{k,m}) = \begin{cases} \dfrac{p(\theta_{k,m}|\theta_k)}{\pi(r^2_{k,\max} - r^2_{k,\min})}; & r_k \in [r_{k,\min}, r_{k,\max}] \\ 0; & \text{otherwise.}\end{cases} \tag{9}$$

The aim is to approximate $f_m(Ht_k; \theta_{k,m})$ by a Gaussian mixture in the surveillance (target position) space,

$$f_m(Ht_k; \theta_{k,m}) \approx \sum_{g=1}^{G_k} \gamma_{k,g}\mathcal{N}(Ht_k; \hat{z}_{k,g}, R_{k,g}), \tag{10}$$

where each element of the Gaussian mixture (10) is termed a “measurement component”. Each measurement component $g = 1, \ldots, G_k$ is defined by its mean $\hat{z}_{k,g}$, covariance $R_{k,g}$ and relative probability $\gamma_{k,g} \ge 0$, with

$$\sum_{g=1}^{G_k} \gamma_{k,g} = 1. \tag{11}$$

As in Kronhamn (1998), Mušicki (2008a,b), divide range interval $[r_{k,\min}, r_{k,\max}]$ in G_k subintervals in a geometric progression

$$\frac{r_{k,g+1}}{r_{k,g}} = \rho_k; \quad g = 1\cdots G_k, \tag{12}$$

where $r_{k,1} = r_{k,\min}$ and $r_{k,G_k+1} = r_{k,\max}$, and

$$\rho_k = \left(\frac{r_{k,\max}}{r_{k,\min}}\right)^{1/G_k} \tag{13}$$

Define $\Delta r_{k,g} = r_{k,g+1} - r_{k,g}$ and $\bar{r}_{k,g} = 0.5(r_{k,g+1} + r_{k,g})$ as the length and mean range of range subinterval g.

Range subinterval g corresponds to segment g in the surveillance space, defined in polar coordinates centered on the sensor position $z_{k,s}$ by $r_{k,g}, r_{k,g+1}, \theta_{k,m} - \sigma_{k,\theta}, \theta_{k,m} + \sigma_{k,\theta}$. Segment g is approximated by an ellipse whose mean value and covariance are used as mean and covariance of measurement component g respectively, $\hat{z}_{k,g}$ and $R_{k,g}$:

$$\hat{z}_{k,g} = z_{k,s} + \bar{r}_{k,g}\begin{bmatrix}\cos(\theta_{k,m})\\ \sin(\theta_{k,m})\end{bmatrix} \tag{14}$$

$$R_{k,g} = T_{k,m}\begin{bmatrix}(\Delta r_{k,g}/2)^2 & 0\\ 0 & (\bar{r}_{k,g}\sigma_{k,\theta})^2\end{bmatrix}T^T_{k,m}, \tag{15}$$

with rotation matrix $T_{k,m}$ defined by

$$T_{k,m} = \begin{bmatrix}\cos(\theta_{k,m}) & -\sin(\theta_{k,m})\\ \sin(\theta_{k,m}) & \cos(\theta_{k,m})\end{bmatrix}. \tag{16}$$

Departing from Kronhamn (1998), the probability $\gamma_{k,g}$ that segment g contains the target is proportional to the area covered by the measurement component g, as in Mušicki (2008a,b) [1]

$$\gamma_{k,g} = \frac{\sqrt{\det(R_{k,g})}}{\sum_{h=1}^{G_k}\sqrt{\det(R_{k,g})}} = \rho_k^{2g-2}\frac{\rho^2_k - 1}{\rho^{2G_k}_k - 1}. \tag{17}$$

The true measurement component at time k is the measurement g defined by $r_k \in [r_{k,g}, r_{k,g+1}]$.

One Gaussian mixture measurement model of 6 components is presented in Fig. 1, where each measurement component is

[1] Second equation provided by anonymous reviewer.

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