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Brief paper

Robust PID controller tuning based on the heuristic Kalman algorithm[☆]

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ABSTRACT

This paper presents a simple but effective tuning strategy for robust PID controllers satisfying multiple $\mathcal{H}_\infty$ performance criteria. Finding such a controller is known to be computationally intractable via the conventional techniques. This is mainly due to the non-convexity of the resulting control problem which is of the fixed order/structure type. To solve this kind of control problem easily and directly, without using any complicated mathematical manipulations and without using too many "user defined" parameters, we utilize the heuristic Kalman algorithm (HKA) for the resolution of the underlying constrained non-convex optimization problem. The resulting tuning strategy is applicable both to stable and unstable systems, without any limitation concerning the order of the process to be controlled. Various numerical studies are conducted to demonstrate the validity of the proposed tuning procedure. Comparisons with previously published works are also given.

1. Introduction

It is a matter of fact that the PID (proportional-integral-derivative) controller is the most widely used in industrial applications. This is mainly due to its ability in solving a broad class of practical control problems as well as its structural simplicity, allowing the operators to use it without much difficulty. In addition, many PID tuning rules have been reported in the literature (see Aström and Hägglund (1995) for a good overview), which are simple and easy to use. However, most of these tuning methods have a limited domain of applications mainly due to restrictive assumptions concerning the process model.

Consequently, developing PID tuning techniques for "arbitrary" process models, satisfying some performance specifications remains an important issue. $\mathcal{H}_\infty$ control theory is a good approach to tackle this problem. Indeed, many robust stability and performance problems can be cast and solved into the $\mathcal{H}_\infty$ framework, without any limitation in the order of the plant. However, the order of the controller thus obtained is almost always greater than or equal to that of the process. This is of course unacceptable for a correct implementation with most of the commercially available PID controllers (Grassi & Tsakalis, 2000). In these conditions, the design step must take into account the structure of the controller.

Unfortunately, the problem of designing a robust controller with a given fixed structure (e.g. a PID) remains an open issue. This is mainly due to the fact that the set of all fixed order/structure stabilizing controllers is non-convex and disconnected in the space of controller parameters. This is a major source of computational intractability and conservatism (Rockafellar, 1993). Nevertheless, due to their practical importance, some new approaches for structured control have been proposed in the literature. Most of them are based on the resolution of Linear Matrix Inequalities LMIs (see for instance: Apkarian, Noll, and Duong Tuan (2003), Cao, Lam, and Sun (1998), Ebihara, Tokuyama, and Hagiwara (2004), Genc (2000), Grigoriadis and Skelton (1994), He and Wang (2006), Iwasaki and Skelton (1995), Mattei (2000), and Saeki (2006)). However, a major drawback with these kinds of approaches is the use of Lyapunov variables, whose number grows quadratically with the system size. For instance, if we consider a system of order 70, this requires, at least, the introduction of 2485 unknown variables whereas we are looking for the parameters of a fixed order/structure controller which contains a comparatively very small number of unknowns. It is then necessary to introduce new techniques capable of dealing with the non-convexity of certain problems arising in automatic control without introducing extra unknown variables.

In this spirit, Kim, Maruta, and Sugie (2008) (see also Maruta, Kim, and Sugie (2008)) have proposed to solve the non-convex optimization problem arising in the design of optimal PI/PID controllers, by the use of an augmented Lagrangian particle swarm optimization (ALPSO) (Sedlaczek & Eberhard, 2006). Although the results obtained with this method are very convincing, it seems that the weakness of this approach lie mainly in the large number of parameters which have to be set by the user, namely: the

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number of particles, the initial velocity of the particles and their initial positions, the value of the inertia factor, the value of the cognitive factor, the value of the social factor and the maximum number of iterations. In addition to these latter parameters, some other parameters have to be set by the user to handle the constraints of the optimization problem. The difficulty is that there is no systematic procedure to select correctly the above mentioned parameters. The only way is thus to proceed by trial and error, but this is time consuming and can be very difficult to do for a large number of parameters.

Since the problem of selecting in advance many parameters is not obvious at all, it appears necessary to develop tuning strategies requiring the smallest possible number of "user defined" parameters. For this purpose, it seems interesting to use the heuristic Kalman algorithm (Toscano and Lyonnet, in press-a; in press-b) because it possesses only three "user defined" parameters. The HKA (Heuristic Kalman Algorithm) enters into the category of the so called "evolutionary computation algorithms". It shares with PSO interesting features such as: ease of implementation, low memory and CPU speed requirements, search procedure based only on the values of the objective function, no need of strong assumptions such as linearity, differentiability, convexity etc, to solve the optimization problem. In fact it could be used even when the objective function cannot be expressed in an analytic form, in this case, the objective function is evaluated through simulations.

The main objective of this paper is to develop a simple and easy to use tuning strategy for robust PID controllers satisfying multiple $\mathcal{H}_\infty$ specifications. Finding such controller gain is known to be computationally intractable by the conventional techniques. Therefore, to solve this design problem easily and directly, without using too many "user defined" parameters, we utilize the heuristic Kalman algorithm for the resolution of the underlying constrained non-convex optimization problem. The resulting tuning method is applicable both to stable and to unstable systems, without any limitation concerning the order of the process to be controlled. However, it is difficult to guarantee its effectiveness in a theoretical way, because, as the PSO, HKA is essentially a stochastic method. Nevertheless, we have evaluated the effectiveness of the proposed method, empirically, through various numerical experiments.

The remaining part of this paper is organized as follows. In Section 2, the robust PID controller design based on the heuristic Kalman algorithm (HKA), is presented. Section 3 shows the validity of the proposed approach on various numerical applications, comparisons with previously published works are also given. Finally, Section 4 concludes this paper.

2. Robust PID controller design based on the heuristic Kalman algorithm (HKA)

In this section, a practical design procedure to determine the PID tuning parameters is presented. To this end, we first formulate the problem of designing a robust PID controller as an optimization problem.

2.1. Formulation of the optimization problem

Consider the general feedback setup shown in Fig. 1, in which $G(s)$ represents the transfer matrix of the generalized process to be controlled

$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s)\begin{bmatrix} w \\ u \end{bmatrix}, \quad \text{with: } G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{1}$$

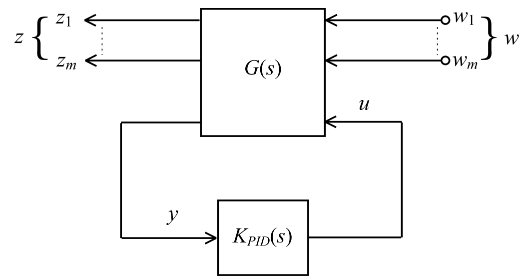

Fig. 1. Block diagram of the PID feedback control system.

and $K_{PID}(s)$ is the transfer matrix of the PID controller

$$K_{PID}(s) = K_p + K_i\frac{1}{s} + K_d\frac{s}{1+\tau s} = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$

$$\text{with: } \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] = \left[\begin{array}{cc|c} 0 & 0 & K_i \\ 0 & -\frac{1}{\tau}I & -\frac{1}{\tau^2}K_d \\ \hline I & I & K_p + \frac{1}{\tau}K_d \end{array}\right] \tag{2}$$

where $K_p \in \mathbf{R}^{n_u \times n_y}$ is the proportional gain, $K_i \in \mathbf{R}^{n_u \times n_y}$ and $K_d \in \mathbf{R}^{n_u \times n_y}$ are the integral and derivative gains respectively, and τ is the time constant of the filter applied to the derivative action.

This low-pass first-order filter ensures the properness of the PID controller and thus its physical realizability. In addition, since $G(s)$ is strictly proper (i.e. it is assumed that $D_{22} = 0$), the properness of the controller ensures the well-posedness of feedback loop.

As depicted Fig. 1, the closed-loop system has m external input vectors $w_1 \in \mathbf{R}^{n_{w_1}}, \ldots, w_m \in \mathbf{R}^{n_{w_m}}$ and m output vectors $z_1 \in \mathbf{R}^{n_{z_1}}, \ldots, z_m \in \mathbf{R}^{n_{z_m}}$. Roughly speaking, the global input vector $w = [w_1 \;\cdots\; w_m]^{\mathrm{T}}$ captures the effects of the environment on the feedback system; for instance noise, disturbances and references. The global output vector $z = [z_1 \;\cdots\; z_m]^{\mathrm{T}}$ contains all characteristics of the closed-loop system that are to be controlled. To this end, the PID control law $K_{PID}(s)$, utilizes the measured output vector $y \in \mathbf{R}^{n_y}$, to elaborate the control action vector $u \in \mathbf{R}^{n_u}$ which modify the natural behavior of the process $G(s)$.

The objective is then to determine the PID parameters (K_p, K_i, K_d, τ) allowing to satisfy some performance specifications such as: a good set point tracking, a satisfactory load disturbance rejection, a good robustness to model uncertainties and so one. A powerful way to enforce these kinds of requirements is first to formulate the performance specifications as an optimization problem and then to solve it by an appropriate method. In the $\mathcal{H}_\infty$ framework, the optimization problem can take one of the following forms:

$$\begin{array}{ll} \min & J_\infty(x) = \|T_{w_1z_1}(s,x)\|_\infty \\ \text{s.t.} & g_1(x) = \arg\max\limits_{\lambda_i(x)}\{\mathrm{Re}(\lambda_i(x)),\ \forall i\} - \lambda_{\min} \leqslant 0 \\ & g_2(x) = \|T_{w_2z_2}(s,x)\|_\infty - \gamma_2 \leqslant 0 \\ & \vdots \\ & g_m(x) = \|T_{w_mz_m}(s,x)\|_\infty - \gamma_m \leqslant 0 \end{array} \tag{3}$$

or also:

$$\begin{array}{ll} \min & J_\lambda(x) = \arg\max\limits_{\lambda_i(x)}\{\mathrm{Re}(\lambda_i(x)),\ \forall i\} \\ \text{s.t.} & g_1(x) = \|T_{w_1z_1}(s,x)\|_\infty - \gamma_1 \leqslant 0 \\ & g_2(x) = \|T_{w_2z_2}(s,x)\|_\infty - \gamma_2 \leqslant 0 \\ & \vdots \\ & g_m(x) = \|T_{w_mz_m}(s,x)\|_\infty - \gamma_m \leqslant 0 \end{array} \tag{4}$$

where $T_{w_iz_i}(s,x)$ denotes the closed-loop transfer matrix from w_i to z_i, $x \in \mathbf{R}^{n_x}$ is the vector of decision variables regrouping the entries of the matrices K_p, K_i, K_d and the time constant τ: $x =$