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Brief paper

Limitations of nonlinear discrete periodic control for disturbance attenuation and robust stabilization ☆

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Abstract

This paper presents a performance analysis of nonlinear periodically time-varying discrete controllers acting upon a linear time-invariant discrete plant. Time-invariant controllers are distinguished from strictly periodically time-varying controllers. For a given nonlinear periodic controller, a time-invariant controller is constructed. Necessary and sufficient conditions are given under which the time-invariant controller gives strictly better control performance than the time-invariant controller from which it was obtained, for the attenuation of l_p exogenous disturbances and the robust stabilization of l_p unstructured perturbations, for all $p \in [1, \infty]$.

Keywords: Disturbance attenuation; Robust stabilization; Discrete nonlinear periodic control; l_p space

1. Introduction

The advantages and limitations of time-varying linear and nonlinear feedback control have been actively researched for the past two decades. Linear time-varying (LTV) controllers have been shown to have advantages over linear time-invariant (LTI) controllers in a number of important control problems such as the improvement of phase and gain margins and asymptotic stabilization (c.f. Allwright, Astolfi, & Wong, 2005; Das & Rajagopalan, 1992; Khargonekar, Poolla, & Tannenbaum, 1985; Moreau & Aeyels, 2004).

In this paper we consider the use of nonlinear periodic discrete control for the problems of disturbance attenuation and robust stabilization of a LTI discrete plant. The problem of disturbance attenuation consists of finding a controller which stabilizes the plant and minimizes the effect of the disturbance input on the disturbance output. The problem of robust stabilization of an LTI plant involves considering a family of plants, and obtaining a controller which stabilizes the closed loop system for all plants in the family. Numerous authors have shown that linear and nonlinear time-varying (NLTV) controllers offer no advantages over LTI controllers for these problems (e.g. Chapellat & Dahleh, 1992; Poolla & Ting, 1987; Schmid & Zhang, 2001; Shamma & Dahleh, 1991; Zhang & Zhang, 2000).

In this paper, we begin by considering a nonlinear strictly periodically time-varying discrete system (periodic with period $N \geqslant 2$) G and obtain a time-invariant discrete system G_{TI} by averaging it. Necessary and sufficient conditions are presented under which G_{TI} has strictly smaller induced system norm than G. This result is used to compare the performance of strictly periodically time-varying controllers and time invariant controllers for the problems of disturbance attenuation and robust stabilization. The analysis distinguishes time-invariant and strictly periodically time-varying controllers. For a given strictly periodically time-varying controller of arbitrary period $N \geqslant 2$, a time-invariant controller will be constructed and compared with the given strictly periodically time-varying controller.

Firstly, we give conditions under which the constructed time-invariant controller gives strictly better disturbance attenuation performance than the given strictly periodically time-varying controller. This extends the analysis of discrete linear controllers in Schmid and Zhang (2001) to nonlinear discrete

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controllers. For sampled-data systems, similar results on nonlinear controllers were given in Schmid and Zhang (2003). Secondly, we give conditions under which the constructed time-invariant controller yields strictly greater additive and multiplicative robust stability margins than the given strictly periodically time-varying controller.

The results imply that the use of strictly periodically time-varying controllers for achieving certain performance specifications may come at a price; a strictly periodic controller can give inferior performance for the attenuation of exogenous disturbances and the robust stabilization of unstructured perturbations, relative to that achievable by a time-invariant controller.

This paper is organized as follows. Section 2 lists some standard definitions and results. Section 3 considers a strictly periodically time-varying system and compares its induced system norm with that of the time-invariant system obtained by averaging it. In Section 4 we give a performance analysis of strictly periodically time-varying controllers for the classical problems of disturbance attenuation and robust stabilization. In Section 5 we give the proof of the main result from Section 3. Section 6 applies the results to an example, and Section 7 contains some concluding remarks.

2. Mathematical preliminaries

For any $p \in [1, \infty]$ and positive integer n, l_p^n denotes the discrete signal space of signals $u : \mathbf{Z}^+ \to \mathbf{R}^n$, equipped with the usual norm $\| \cdot \|_p$. We consider systems $G : l_p^n \to l_p^m$. G is assumed to be nonlinear, in the sense of not necessarily linear.

For any $\tau \in \mathbf{Z}$ and any $p \in [1, \infty]$, let $q^{-\tau} : l_p^n \to l_p^n$ be the back shift operator defined by

$$(q^{-\tau} u)(t) = u(t - \tau), \tag{1}$$

where zeroes padding is assumed. For any $u \in l_p^n$, and for any $\tau \in \mathbf{Z}$, $\|q^{-\tau} u\|_p = \|u\|_p$. Let $P_\tau : l_p^n \to l_p^n$ be the truncation operator defined by

$$(P_\tau u)(t) = \begin{cases} u(t) & \text{if } t \leqslant \tau, \\ 0 & \text{elsewhere.} \end{cases} \tag{2}$$

G is *causal* if, for all $\tau \in \mathbf{Z}$ and all $u \in l_p^n$, $P_\tau G u = P_\tau G P_\tau u$. G is *strictly causal* if, for all $\tau \in \mathbf{Z}$ and all $u \in l_p^n$, $P_\tau G u = P_\tau G P_{\tau-1} u$. G has *pointwise finite memory* (Shamma & Zhao, 1993) if there exists a function $FM(\cdot, \cdot; G) : l_p^n \times \mathbf{Z}^+ \to \mathbf{Z}^+$ such that for all $u \in l_p^n$ and $t \in \mathbf{Z}^+$,

(1) $FM(u, t; G) \geqslant t$,
(2) $FM(u, t; G) = FM(P_t u, t; G)$,
(3) $(I - P_{FM(u,t;G)}) G u = (I - P_{FM(u,t;G)}) G (I - P_t) u$.

G has *pointwise fading memory* if it can be approximated arbitrarily closely in norm by pointwise finite memory systems. G is *time invariant* if $G = qGq^{-1}$. G is *periodically time-varying* with period $N \geqslant 1$ if $G = q^N G q^{-N}$, $G \neq q^\tau G q^{-\tau}$, $\forall 1 \leqslant \tau \leqslant N-1$. G is *strictly periodically time-varying* if $N \geqslant 2$. Thus strictly periodically time-varying systems are distinguished from time invariant systems. For brevity strictly periodically time-varying systems will be referred to as *strictly periodic* systems. The l_p-induced *norm* of G is

$$\|G\|_p = \sup \left\{ \frac{\|Gu\|_p}{\|u\|_p} : u \in l_p^n, u \neq 0 \right\}. \tag{3}$$

G is l_p *finite gain stable* if $\|G\|_p < \infty$. For stable G, there exists a sequence of non-zero signals $\{u_k\}_{k \in \mathbf{Z}^+} \subseteq l_p^n$ such that $\|G\|_p = \lim_{k \to \infty} \|Gu_k\|_p / \|u_k\|_p$; we say G *attains its norm* on the sequence. The *incremental norm* of G is

$$\|G\|_p^{\text{inc}} = \sup \left\{ \frac{\|Gu - Gv\|_p}{\|u - v\|_p} : \ u, v \in l_p^n, \ u - v \neq 0 \right\}. \tag{4}$$

G is *incrementally l_p finite gain stable* if $\|G\|_p^{\text{inc}} < \infty$. For linear G, the norm and incremental norm agree. If G has period $N \geqslant 2$, then for each $0 \leqslant \tau \leqslant N-1$, the system $G_\tau : l_p^n \to l_p^m$ with $G_\tau = q^\tau G q^{-\tau}$ also has period N. The system $G_{\text{TI}} : l_p^n \to l_p^m$ with

$$G_{\text{TI}} = \frac{1}{N} \sum_{\tau=0}^{N-1} G_\tau \tag{5}$$

is a time-invariant system, because

$$q^1 G_{\text{TI}} q^{-1} = \frac{1}{N} \sum_{\tau=0}^{N-1} q^1 G_\tau q^{-1} = \frac{1}{N} \sum_{\tau=1}^{N} G_\tau = G_{\text{TI}}.$$

If G is (incrementally) l_p finite gain stable then so is G_τ, for each $1 \leqslant \tau \leqslant N-1$, and also G_{TI}. We denote $c_0^n = \{u \in l_\infty^n : \lim_{t \to \infty} |u(t)|_\infty = 0\} \subseteq l_\infty^n$, and use $b_p^n(r)$ to denote the closed ball in l_p^n of radius r. For $p \in [1, \infty]$, we say that $S = \{y_1, y_2, \ldots, y_N\} \subseteq l_p^n$ is an l_p *strictly convex set* if and only if

$$\left\| \frac{1}{N} \sum_{i=1}^{N} y_i \right\|_p < \max\{\|y_i\|_p : 1 \leqslant i \leqslant N\}. \tag{6}$$

For $p \in (1, \infty)$, the spaces l_p^n are *strictly convex*, and this implies that a set S is an l_p strictly convex set if and only if it contains at least two elements. For $p = 1$ and ∞, we obtain conditions under which S is l_p strictly convex in Section 5. For $p \in (1, \infty)$, the spaces l_p^n are also *uniformly convex* and satisfy the following property: for some integer $N \geqslant 2$ and for some $r > 0$, let $S = \{y_1, y_2, \ldots, y_N\} \subseteq b_p^n(r)$ be such that there exist $y_j, \ y_k \in S$ and $\delta > 0$ satisfying $\|y_j - y_k\|_p > \delta$. Then there exists an $\varepsilon(\delta, r) > 0$ such that

$$\max\{\|y_i\|_p : 1 \leqslant i \leqslant N\} - \left\| \frac{1}{N} \sum_{i=1}^{N} y_i \right\|_p > \varepsilon(\delta, r). \tag{7}$$

3. Analysis of discrete periodic system norms

In this section we consider a finite gain stable strictly periodic system G with period $N \geqslant 2$. The time-invariant system G_{TI} derived by averaging G as in (5) is such that $\|G_{\text{TI}}\|_p \leqslant \|G\|_p$