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Study on initial stage of capillary rise dynamics

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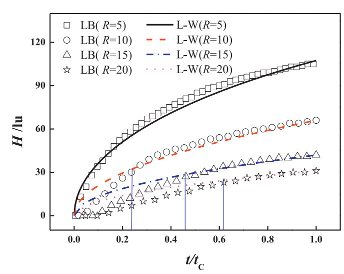
HIGHLIGHTS

- Capillary rise dynamics is investigated using multiphase lattice Boltzmann method.
- Various effects on the initial differences of capillary rise are analyzed.
- The capillary rise height is proportional initially to the square of the time.
- The numerical results give quantitative agreement with experimental data.

GRAPHICAL ABSTRACT

Capillary rise dynamics in a capillary tube was investigate using multiphase lattice Boltzmann method. The effects of gravity, adhesion, surface tension, viscous drag, inertia, and the tube radius on the initial differences are analyzed in detail to identify the mechanisms for the differences.



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ABSTRACT

A multiphase lattice Boltzmann method (LBM) was developed to investigate capillary rise dynamics in a capillary tube. The numerical results give quantitative agreement with experimental data when inertia, the dynamic contact angle, and the entrance effect were taken into account. The results show that the capillary rise can be divided into an initial stage where the capillary rise dynamics significantly deviate from the well-known Lucas–Washburn (L–W) law, $H \sim t^{1/2}$, and a second stage where, however, approaches the predictions of the L–W law. The effects of gravity, adhesion, surface tension, viscous drag, inertia, and the tube radius on the initial differences are analyzed in detail to identify the mechanisms for the differences.

1. Introduction

The capillary rise is an important phenomenon that has a wide range of practical applications, such as in oil recovery, civil engineering, dyeing of textile fabrics, ink printing, and a variety of other fields. The capillary rise also provides the appropriate framework for studying the dynamic wetting of a fluid on a solid surface, which has attracted much research interest.

The first theoretical analyses of capillary rise were conducted by Lucas [1] and Washburn [2], with their model known as the Lucas–Washburn (L–W) equation, in which the process was

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Nomenclatures

Bo	bond number
c_α	lattice speed along the α direction
D	capillary tube diameter
$\mathbf{e}_\alpha$	lattice velocity vector
f_α	density distribution function along the α direction
f^{eq}	equilibrium density distribution function
F	fluid–fluid interaction force
F_{ads}	solid–fluid interaction force
F_Y	Young's force
G	interaction strength
G_{ads}	adsorption coefficient
H	liquid column height
$\tilde{;} H$	non-dimensional capillary rise height
p	pressure
R	universal gas constant, capillary tube radius
r	droplet or bubble radius
R^2	correlation coefficient
s	switch parameter
t	time
T	temperature
$\mathbf{u}$	velocity
u^{eq}	modified velocity
w	weight
x, y	x coordinate and y coordinate

Greek Symbols

Δ	difference
ψ	interaction potential
θ	contact angle
θ_0	equilibrium contact angle
σ	surface tension
τ	collision-relaxation time
μ	dynamic viscosity
ν	kinetic viscosity
ρ	density
ψ_0, ρ_0	arbitrary constants in Eq. (5)

Subscripts

α	lattice direction
d	dynamic
l,v	liquid, vapor
max	maximum value
t	total
0	equilibrium

described by the balance between the capillary force, viscous force and gravity. The L–W equation predicts that the capillary rise is proportional to the square root of time, which is referred as the L–W law [3–5]. However, the L–W law is not always satisfied experimentally, especially in microgravity [6–11] mainly due to the following three well known reasons. Firstly, the L–W equation fails to predict the initial stage of the meniscus rise due to the lack of an inertial term [6,12–15]. Secondly, the equation is based on Poiseuille flow across the capillary tube with a parabolic velocity profile, which cannot apply to the tube entrance region [16–18]. Finally, the contact angle is assumed to be invariable during the entire capillary rise process; however, the contact angle is known to depend on the speed of the advancing meniscus, leading to the concept of the dynamic contact angle, θ_d [19–21]. Therefore, experimental observations have led to some corrections as $H{\sim}t$ [6–8] or $H{\sim}t^2$ [9–12] to describe the initial stage of the capillary rise, as shown in Table 1.

There are many multiphase/multi-component flow models in the lattice Boltzmann method, such as "color gradient method" of Gunstensent et al. [22], the "pseudo-potential method" of Shan and Chen [23,24] and Shan and Doolen [25], and the "free energy model" of Swift et al. [26], "finite density models" of Luo [27] and He et al. [28], "energy model" of He et al. [29]. Shan–Chen model has numerous shortcomings, such as spurious interface velocities, surface tension first order errors due to a bad discretization. However, for Shan–Chen model, (1) it is easy and flexible to implement; (2) the phase separation or components is automatic which improving significantly numerical efficiency; (3) the Shan–Chen model also improves the isotropy of the surface tension; (4) the phase separation was modeling by inducing a long-range interaction force which mimics the pairwise interaction between different phase/components and has a much clearer microscopic physical scenario; especially, for the capillary rise or wetting problems, both fluid–fluid and solid–fluid interactions should be considered. In present manuscript, the adhesive interactions for solid–fluid were incorporated according to Marty and Chen's model [30]. This implement was an analog to the fluid–fluid interaction used in Shan–Chen model. Consequently, the interactions of fluid–solid and fluid–fluid are consistent and have much clearer physical meanings. Therefore, the Shan–Chen model was widely used to investigate wetting [31–35] or capillary rise problems [36–42]. Raiskinmak et al. [36] reported results of two-phase lattice Boltzmann simulations of the capillary rise dynamics. They demonstrated that the LBM could be used to model the hydrodynamic behavior inside a capillary tube when the tube diameter is large enough. They also presented results for the dependence of the cosine of the dynamic contact angle on the capillary number Ca. The difference between the dynamic contact angle and the static contact angle has a power-law form, with the exponent very close to 3/2 for capillary rise at zero gravity, with more complex behavior in the presence of gravity. Capillary filling was simulated by Fan et al. [37] using the lattice Boltzmann model in a two-dimensional horizontal capillary channel with an ideally smooth wall. Their investigation revealed that during immiscible-fluid displacement, the contact angle between the interface and the channel wall, as well as the contact line velocity, depend on the applied capillary pressure. Capillary filling for multi-phase flows was investigated by Diotallevi et al. [38,39] using lattice Boltzmann models. They compared their numerical results with the L–W law for various liquid and gas phase density ratios and various ratios of the capillary tube size to the interface width. Reservoirey et al. [40] investigated the dynamics of capillary filling using two lattice Boltzmann

Table 1
Review of $H{\sim}t$ relations for capillary rise.

Author	Year	Relations for $H{\sim}t$	Tube	Method
Siegel [6]	1961	$H{\sim}t + H{\sim}t^{1/2}$(LW)	Vertical	Experiment
Petrash et al. [9]	1963	$H{\sim}t^2 + H{\sim}t$	Vertical	Experiment
Ichikawa and Satoda [7]	1994	$H{\sim}t + H{\sim}t^{1/2}$	Horizontal	Experiment + numerical
Dreyer et al. [10]	1994	$H{\sim}t^2 + H{\sim}t + H{\sim}t^{1/2}$	Vertical	Experiment + numerical
Weislogel and Lichter [8]	1998	$H{\sim}t + H{\sim}t^{3/5} + H{\sim}t^{1/2}$	Vertical	Experiment + numerical
Zhmud et al. [13]	2000	$H{\sim}t^2 + H{\sim}t^{1/2}$	Vertical	Experiment + numerical
Stange et al. [11]	2003	$H{\sim}t^2 + H{\sim}t + H{\sim}t^{1/2}$	Vertical	Experiment + numerical