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Brief paper

Tracking and disturbance rejection for fully actuated mechanical systems[☆]

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ABSTRACT

In this paper, we solve the tracking and disturbance rejection problem for fully actuated passive mechanical systems. We assume that the reference signal r and its first two derivatives $\dot{r}$, $\ddot{r}$ are available to the controller and the disturbance signal d can be decomposed into a finite superposition of sine waves of arbitrary but known frequencies and an arbitrary L^2 signal. We combine the internal model principle with the ideas behind the *Slotine–Li* adaptive controller. The internal model-based adaptive controller that we propose causes the closed-loop state trajectories to be bounded, and the tracking error and its derivative to converge to zero, without any prior knowledge of the plant parameters. An important part of our results is that we prove the existence and uniqueness of the state trajectories of the closed-loop system.

1. Introduction

The internal model principle for LTI systems suggests that the dynamic structure of the exosystem must be included in the controller. For example, to eliminate the steady-state error for step reference or disturbance signals, we need integrators in the loop. If an internal model with transfer function $1/(s^2+\omega^2)$ (with suitable multiplicity) is in the feedback loop and the closed-loop system is stable, then we obtain tracking and/or disturbance rejection for sinusoidal reference and disturbance signals of frequency ω, see for example Davison and Goldenberg (1975). If the reference and disturbance signals are periodic, then the internal model principle leads to repetitive control (see for example Hara, Yamamoto, Omata, and Nakano (1988), and Weiss and Häfele (1999)).

The idea of an internal model has been generalized for output regulation of nonlinear systems by Byrnes, Delli Priscoli, and Isidori (1997) and Isidori (1995). In Byrnes et al. (1997) and Isidori (1995), the exogenous signal is generated by an exosystem and the existence of the controller requires the solvability of the Byrnes–Isidori regulator equations. Recent results on the output regulation of nonlinear systems can be found in Byrnes and Isidori (2003), Delli Priscoli (2004), Huang and Chen (2004) and Serrani, Isidori, and Marconi (2001).

In Jayawardhana and Weiss (2005, in press), a simple LTI internal model is used to solve the disturbance rejection problem for passive nonlinear plants. In Jayawardhana and Weiss (in press), the disturbance d is assumed to be of the form $d = d_0 + d_E$, where $d_0 \in L^2([0,\infty), \mathbb{R}^m)$, and d_E is generated by an LTI exosystem (as in (19)). No precise knowledge of the plant parameters is required in Jayawardhana and Weiss (in press). In this paper, the plant is a fully actuated mechanical system with the vector of generalized coordinates denoted by q, which should track a $\mathcal{C}^2$ reference signal r. We combine an LTI controller as in Jayawardhana and Weiss (in press) with a *Slotine–Li* type adaptive controller (see Slotine and Li (1988)) for rejecting a disturbance signal $d = d_0 + d_E$ as in Jayawardhana and Weiss (in press) and for asymptotically tracking r. We assume that the signals r, $\dot{r}$ and $\ddot{r}$ are available to the controller, but the controller does not know the parameters of the plant.

Our construction can be modified to allow the same LTI compensator to be combined with other passivity-based tracking controllers, for example, the passivity-based adaptive tracking controller in Slotine and Li (1989) or the adaptive tracking controller with adaptive friction compensator in Panteley, Ortega, and Gäfvert (1998).

In Scherpen and Ortega (1997), it is shown that by using the *Slotine–Li* controller and by adding to it a high gain proportional block from the tracking error to the input, the L^2 gain from the disturbance to the tracking error can be made arbitrarily small. However, this approach does not assure that the error converges to zero for a disturbance which is not in L^2. For a recent survey on tracking controllers for fully actuated mechanical systems we refer to Sage, de Mathelin, and Ostertag (1999). Results related to those

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in our paper have appeared in Bonivento, Gentili, and Paoli (2004). The controller in Bonivento et al. (2004) uses an adaptive internal model to find the frequencies of the disturbance, with the plant assumed to be known, while we use an adaptive controller to deal with uncertainty in the plant parameters (with the frequencies known).

We believe that our main contribution is to combine an internal model, which is usually considered for time-invariant systems, with the *Slotine–Li* controller, even though the latter leads to a time-varying system. Moreover, we allow an L^2 component in the disturbance signal, which is a new feature, and we are careful to prove the existence and uniqueness of state trajectories for the closed-loop system. We also show that both the tracking error and its time derivative tend to zero.

Our main results are stated and proved in Section 3. Due to the space constraints, we have no space to include simulation results. For this and for a design procedure for the internal model we refer to Jayawardhana (2006).

Notation. Throughout this paper, the inner product on any Hilbert space is denoted by $\langle\cdot,\cdot\rangle$ and $\mathbb{R}_+ = [0,\infty)$. We refer to Khalil (2000) and van der Schaft (2000) for basic concepts on nonlinear systems and on passivity theory. For a finite-dimensional vector x, we use the norm $\|x\| = \left(\sum_n |x_n|^2\right)^{\frac{1}{2}}$ and for matrices, we use the operator norm induced by $\|\cdot\|$ (the largest singular value). For a square matrix A, $\sigma(A)$ denotes the set of its eigenvalues. For any finite-dimensional vector space $\mathcal{V}$ endowed with a norm $\|\cdot\|_{\mathcal{V}}$, the space $L^2(\mathbb{R}_+,\mathcal{V})$ consists of all the measurable functions $f:\mathbb{R}_+\to\mathcal{V}$ such that $\int_0^\infty \|f(t)\|^2_{\mathcal{V}}\mathrm{d}t < \infty$. The square-root of the last integral is denoted by $\|f\|_{L^2}$. For $f\in L^2(\mathbb{R}_+,\mathcal{V})$ and $T>0$, we denote by f_T the truncation of f to $[0,T]$. The space $\mathcal{C}^1(\mathbb{R}^l,\mathbb{R}^p)$ consists of all the continuously differentiable functions $f:\mathbb{R}^l\to\mathbb{R}^p$, while $\mathcal{C}^2(\mathbb{R}_+)$ consists of all the twice continuously differentiable functions $r:\mathbb{R}_+\to\mathbb{R}$.

2. The Slotine–Li controller

Consider the problem of tracking a $\mathcal{C}^2$ reference signal r with the generalized coordinates q of a fully actuated mechanical system, without precise knowledge of the plant parameters. It is known that in the absence of disturbances, the *Slotine–Li* adaptive controller from Slotine and Li (1988) achieves asymptotic tracking of r with bounded state trajectories. In this section, first we show that the *Slotine–Li* feedback law applied to a fully actuated mechanical system produces a time-varying passive system. Using this, we generalize the results of Slotine and Li (1988) by allowing an L^2 disturbance to act on the plant. We show that, in spite of this disturbance, not only does the tracking error e tend to zero (as shown in van der Schaft (2000)) but also its time derivative $\dot{e}$.

We consider a plant $\mathbf{P}$ described by

$$\mathcal{M}(q)\ddot{q} + \mathcal{D}(q,\dot{q})\dot{q} + g(q) = u, \tag{1}$$

which we call a *fully actuated mechanical system*. Such systems often originate from Euler–Lagrange equations for mechanical systems and they have been extensively studied, see Astolfi, Limebeer, Melchiorri, Tornambe, and Vinter (1997) and Ortega, Loría, Nicklasson, and Sira-Ramírez (1998). Here, $q(t)\in\mathbb{R}^n$ is the vector of *generalized coordinates*, $\mathcal{M}(q)$ is self-adjoint and

$$m_1 I \le \mathcal{M}(q) \le m_2 I, \quad \text{where } m_1, m_2 > 0, \tag{2}$$

$g(q)$ is a locally Lipschitz continuous function (which usually represents forces due to the potential energy) and $u(t)\in\mathbb{R}^n$ is the input (usually, forces or torques). The function $\mathcal{M}(\cdot)$ is assumed to be continuously differentiable and $\mathcal{D}(\cdot,\cdot)$ is assumed to be locally Lipschitz continuous. As usual, we denote $\dot{\mathcal{M}}(q,\dot{q}) = \sum_{j=1}^n \frac{\partial \mathcal{M}}{\partial q_j}\dot{q}_j$. The state of this system is the vector $\begin{bmatrix} q \\ \dot{q}\end{bmatrix}$. We assume that $J(q,\dot{q}) = \dot{\mathcal{M}}(q,\dot{q}) - 2\mathcal{D}(q,\dot{q})$ satisfies $J^{\mathrm{T}}(q,\dot{q}) + J(q,\dot{q}) \le 0$, so that

$$\left\langle \left(\frac{1}{2}\dot{\mathcal{M}} - \mathcal{D}\right)a, a\right\rangle \le 0 \quad \forall a\in\mathbb{R}^n. \tag{3}$$

We remark that if $g(q) = (\nabla V(q))^{\mathrm{T}}$, where $V\in\mathcal{C}^1(\mathbb{R}^n,\mathbb{R}_+)$ is called the *potential energy*, then the plant $\mathbf{P}$ with output signal $\dot{q}$ is *passive* with respect to the storage function $H(q,\dot{q}) = \frac{1}{2}\langle\mathcal{M}(q)\dot{q},\dot{q}\rangle + V(q)$, i.e., if a state trajectory exists then $\dot{H}\le\langle\dot{q},u\rangle$. We mention that if $J^{\mathrm{T}}+J=0$ then this system is *energy preserving*, meaning that $\dot{H} = \langle\dot{q},u\rangle$.

We assume that $r\in\mathcal{C}^2(\mathbb{R}_+,\mathbb{R}^n)$ and the signals $r,\dot{r},\ddot{r}$ are available to the controller. The input signal u is the sum of a disturbance signal d and the control input s (generated by the controller that we shall design), see Fig. 2(a). We assume that $\mathcal{M}$, $\mathcal{D}$ and g are not known exactly, but we can express them in terms of unknown real parameters $\theta_1,\theta_2,\ldots,\theta_m$ as follows:

$$\left.\begin{aligned} \mathcal{M}(q) &= \sum_{i=1}^m \mathcal{M}_i(q)\theta_i + \mathcal{M}_0(q),\\ \mathcal{D}(q,\dot{q}) &= \sum_{i=1}^m \mathcal{D}_i(q,\dot{q})\theta_i + \mathcal{D}_0(q,\dot{q}),\\ g(q) &= \sum_{i=1}^m g_i(q)\theta_i + g_0(q), \end{aligned}\right\} \tag{4}$$

where $\mathcal{M}_i$ is of class $\mathcal{C}^1$ and $\mathcal{D}_i$, g_i are locally Lipschitz continuous. For any $q,q_1,a,b\in\mathbb{R}^n$, we introduce the matrix $\Phi(q,q_1,a,b)\in\mathbb{R}^{n\times m}$ such that

$$\begin{aligned}\Phi(q,q_1,a,b)\theta &= \left(\sum_{i=1}^m \mathcal{M}_i(q)\theta_i\right)a + \left(\sum_{i=1}^m \mathcal{D}_i(q,q_1)\theta_i\right)b\\ &\quad + \sum_{i=1}^m g_i(q)\theta_i, \end{aligned} \tag{5}$$

where $\theta = \begin{bmatrix}\theta_1 & \theta_2 & \cdots & \theta_m\end{bmatrix}^{\mathrm{T}}$ is the parameter vector.

We describe a first feedback loop which is based on the *Slotine–Li* controller and which eliminates r from the picture, so that the problem is reduced to the input disturbance rejection problem. We denote by $\hat{\mathcal{M}}(q)$, $\hat{\mathcal{D}}(q,\dot{q})$ and $\hat{g}(q)$ the estimates of $\mathcal{M}(q)$, $\mathcal{D}(q,\dot{q})$ and $g(q)$ corresponding to the estimate $\hat{\theta}$ of the unknown parameter vector θ. (This means that $\hat{\mathcal{M}}(q)$ is obtained from (4) by replacing θ with $\hat{\theta}$, and similarly for $\hat{\mathcal{D}}(q,\dot{q})$ and $\hat{g}(q)$.) Consider the feedback law

$$u = \hat{\mathcal{M}}\dot{\xi} + \hat{\mathcal{D}}\xi + \hat{g} + v, \tag{6}$$

where

$$\xi := \dot{r} + \Lambda(r-q), \quad \Lambda = \Lambda^{\mathrm{T}} \ge \mu I > 0, \tag{7}$$

and v is the new input signal, containing d and any other components of the control input z (to be designed). The estimated parameters $\hat{\theta}$ evolve according to

$$\dot{\hat{\theta}} = -\lambda\Phi(q,\dot{q},\dot{\xi},\xi)^{\mathrm{T}}\zeta, \tag{8}$$

where $\zeta = \dot{q} - \xi$ and $\lambda\in\mathbb{R}^{m\times m}$, $\lambda = \lambda^{\mathrm{T}} > 0$, see Fig. 1. Substituting (6) into (1) gives

$$\begin{aligned}\mathcal{M}(q)\dot{\zeta} + \mathcal{D}(q,\dot{q})\zeta &= \left[\hat{\mathcal{M}}(q) - \mathcal{M}(q)\right]\dot{\xi}\\ &\quad + \left[\hat{\mathcal{D}}(q,\dot{q}) - \mathcal{D}(q,\dot{q})\right]\xi + \hat{g}(q) - g(q) + v. \end{aligned}\tag{9}$$

Introducing the estimation error $\tilde{\theta} = \hat{\theta} - \theta$, we have $\hat{\mathcal{M}}(q) - \mathcal{M}(q) = \sum_{i=1}^m \mathcal{M}_i(q)\tilde{\theta}$, and we have similar formulas for $\hat{\mathcal{D}}(q,\dot{q}) - \mathcal{D}(q,\dot{q})$ and $\hat{g}(q) - g(q)$. Now using (5), the formula (9) becomes

$$\mathcal{M}(q)\dot{\zeta} + \mathcal{D}(q,\dot{q})\zeta = \Phi(q,\dot{q},\dot{\xi},\xi)\tilde{\theta} + v. \tag{10}$$