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Brief paper

Controller design for flexible structure vibration suppression with robustness to contacts☆

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ABSTRACT

Model-based feedback control of vibration in flexible structures can be complicated by the possibility that interaction with an external body occurs. If not accounted for, instability or poor performance may result. In this paper, a method is proposed for achieving robust vibration control of flexible structures under contact. The method uses robust linear state feedback, coupled with a state estimation scheme utilizing contact force measurement. Uncertain contact characteristics are modelled by a sector-bounded non-linear function, such that state feedback gains can be synthesized using a matrix inequality formulation of the Popov stability criterion. A separation theorem is used to establish a robust $\mathcal{H}_2$ cost bound for the closed loop system. Experimental results from a multi-mode flexible structure testbed confirm that vibration attenuation and stability can be maintained over a broad range of contact characteristics, in terms of compliance and clearance.

1. Introduction

The design of model-based controllers for vibration suppression may be complicated by the possibility that the structure under control makes contact with an external body. If this interaction has not been considered in the dynamic modelling of the system, then instability or poor control performance may result. This issue is pertinent to a variety of engineering systems, including large moveable structures such as space structures, bridges, telescopes; flexible manipulators or robots mounted on flexible structures; flexible rotor systems with active elements e.g. magnetic bearings; and active vibration isolation systems with flexible structures. With such systems, deflection of the structure may be constrained by actuators with limited stroke, machine components with limited clearance or contact with external bodies.

In structural vibration control, linear multi-variable design techniques can account for some types of model uncertainty. For example, the use of frequency domain uncertainty bounds in $\mathcal{H}_\infty$ designs can account for unmodelled dynamics, as applied to a smart structure satellite by Moreira, Roberto de Franca Arruda, and Inman (2001). $\mathcal{H}_\infty$ designs have also been applied to civil engineering structures, with sensor and actuator locations chosen to give additional robustness to neglected modes (Kar, Seto, & Doi, 2000). Structured uncertainty models can give better performance-robustness trade-offs, particularly when uncertainty in natural frequencies and damping of flexural modes must be dealt with Balas and Doyle (1994) and Boulet, Francis, Hughes, and Hong (1997). Unfortunately, these methods are not suited to structures with contact constraints, as the contact characteristics can be both uncertain and non-linear. Moreover, if the contact is stiff compared with the compliance of the structure, then the open loop dynamics can change markedly under contact and robust stability requirements will be severe.

Certain symmetric controller designs have attractive properties when applied to flexible structures with collocated actuators and sensors (Arbel & Gupta, 1981; Fujisake, Ikeda, & Miki, 2001; Nagashio & Kida, 2004). Closed loop stability follows from positive definiteness of the model matrices, and is independent of parameter values. Symmetric controllers giving optimal linear quadratic (Nagashio & Kida, 2004) and $\mathcal{H}_\infty$ (Arbel & Gupta, 1981) performance have both been derived. In some systems, however, collocation is not an option, and even when it is, there may be benefits to be gained, in terms of performance or fault tolerance, by using additional sensors or actuators. Thus, there is good motivation for developing design methods that can cope with non-collocation.

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When structures are excited by broadband disturbances, quadratic measures of vibration attenuation are often appropriate, and so there have been recent efforts to develop robust $\mathcal{H}_2$ controllers for uncertain linear systems, particularly with real parameter uncertainty. Methods of controller synthesis based on the Popov criterion and related Lur'e Postnikov Lyaponov function have proved attractive, as stability multipliers can be used to establish tight bounds on robust stability and performance (Haddad & Bernstein, 1995). Popov controller synthesis has been used successfully on aerospace structures to achieve robust performance with uncertain natural frequencies (How, Collins, & Haddad, 1994; How, Hall, & Haddad, 1994).

This paper applies a Popov-based controller synthesis to a class of system where contact between a flexible structure and an external body can be modelled by a sector-bounded static non-linearity. A linear state feedback design with a robust $\mathcal{H}_2$ cost bound is synthesised, using a matrix inequality formulation of the Popov stability criterion. The output feedback controller employing observer based state feedback possesses separation properties that can be used to establish a $\mathcal{H}_2$ cost bound. The controller requires inputs of measured contact force, and one or more vibration variables. The method places no restrictions on the number of sensor or actuators and does not require collocation. Experimental results from a multi-mode flexible structure testbed are presented.

2. System model

The dynamics of a flexible structure constrained by a single contact can be described by a finite-dimensional approximation as

$$M\ddot{\zeta} + C\dot{\zeta} + K\zeta = E_d d - E_c f + E_u u$$
$$z = G_c\zeta, \qquad y = G_y\zeta$$
$$f = \phi(z) \in \Phi. \tag{1}$$

The vector $\zeta \in \mathbb{R}^n$ contains n displacement states for the structure, $d \in \mathbb{R}^{n_d}$ represents disturbance forces acting on the structure and $u \in \mathbb{R}^{n_u}$ are the actuation signals. The system output $y \in \mathbb{R}^{n_y}$, comprises measurable states available for control feedback. Contact is assumed to occur at a single known location, such that the contact force f is a function only of the displacement at the contact location z. The minus sign is used before E_c as f generally acts in the opposite direction to z. The matrix $M \in \mathbb{R}^{n\times n}$ is the mass matrix, $C \in \mathbb{R}^{n\times n}$ represents viscous damping forces, and the stiffness matrix $K \in \mathbb{R}^{n\times n}$ models the structural compliance. Parameter values can be derived by standard methods, such as finite element (FE) modelling, system identification and FE model-updating.

In state space, the open loop system model follows as

$$\text{SystemA}: \quad \begin{aligned} &\dot{x} = Ax + B_d d - B_c f + B_d u \\ &z = C_c x, \qquad y = C_y x \\ &f = \phi(z) \in \Phi. \end{aligned} \tag{2}$$

Taking the state vector as $x = [\zeta^{\mathrm{T}}\ \dot{\zeta}^{\mathrm{T}}]^{\mathrm{T}}$ then

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$$
$$B_d = \begin{bmatrix} 0_{n\times n_d} \\ M^{-1}E_d \end{bmatrix}, \qquad B_c = \begin{bmatrix} 0_{n\times 1} \\ M^{-1}E_c \end{bmatrix}, \qquad B_u = \begin{bmatrix} 0_{n\times n_u} \\ M^{-1}E_u \end{bmatrix}$$
$$C_c = \begin{bmatrix} G_c & 0_{1\times n} \end{bmatrix}, \qquad C_y = \begin{bmatrix} G_y & 0_{n_y\times n} \end{bmatrix}. \tag{3}$$

The uncertain contact force is assumed to be a function of the local displacement state $f = \phi(z) \in \Phi$, satisfying

$$\Phi \triangleq \{\phi(z) : 0 \le z\phi(z) \le kz^2\}. \tag{4}$$

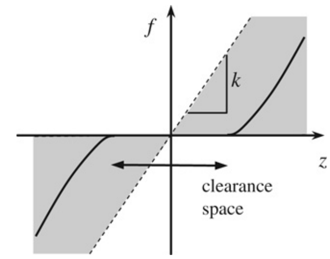

Fig. 1. Sector-bound contact force model: encompasses typical nonlinear stiffness characteristics with clearance.

This sector-bound condition (equivalent to $f(f - kz) \le 0$) can accommodate a range of non-linear contact force characteristics, including the finite clearance example in Fig. 1. The condition is also applicable when the clearances between the structure and the contact surfaces are uncertain, or slowly changing. Following a brief statement of notation and formulas, the remainder of the paper is divided into two main parts. The first part covers the controller design method, while the second part presents results from modelling and experiment involving a multi-mode testbed.

2.1. Notation

E, **tr** — expectation value, trace
$A < 0, A \le 0$ — negative definiteness, semi-definiteness of symmetric matrix A
$I_{n\times m}$ — $n \times m$ identity matrix
$\mathrm{diag}(a_1, \ldots, a_n)$ — denotes the $n\times n$ diagonal matrix with diagonal elements $a_1, \ldots, a_n$.

Lemma 1. *Schur complement formula:*

$$\begin{bmatrix} X & N^{\mathrm{T}} \\ N & Y \end{bmatrix} < 0 \Leftrightarrow X - N^{\mathrm{T}}Y^{-1}N < 0, \quad Y < 0$$

where X and Y are symmetric matrices.

Lemma 2. $X < 0 \Leftrightarrow RXR^{\mathrm{T}} < 0$ *where R is any square nonsingular matrix of compatible dimensions.*

3. Controller design method

3.1. Robust performance criterion

Consider a general closed loop system model appropriate to linear feedback control of System A:

$$\text{SystemB}: \quad \begin{aligned} &\dot{x} = \mathcal{A}x + \mathcal{B}_d d - \mathcal{B}_c f \\ &z = \mathcal{C}_c x, \qquad w = \mathcal{C}_w x \\ &f = \phi(z) \in \Phi. \end{aligned} \tag{5}$$

The output w comprises signals to be regulated. With a dynamic controller of order n_c then $x \in \mathbb{R}^{2n+n_c}$.

A bound on robust performance can be established for System B, based on an impulse-response interpretation of the $\mathcal{H}_2$ norm (Paganini & Feron, 1997), evaluated as the worst-case energy of the output w for impulse input $d = d_0\delta(t)$ averaged over all random vectors having covariance $\mathbf{E}(d_0^{\mathrm{T}}d_0) = I$. Adopting a Lur'e type Lyapunov function candidate (Haddad & Bernstein, 1995; Popov, 1961):

$$V(x) = x^{\mathrm{T}}Px + 2\nu\int_0^z \phi(\lambda)\mathrm{d}\lambda. \tag{6}$$

$P = P^{\mathrm{T}} > 0$ and $\nu \ge 0$ must be found, such that

$$\dot{V}(x) + w^{\mathrm{T}}w < 0 \quad \forall x \ne 0, \phi(z) \in \Phi. \tag{7}$$